NAME: **2017**

Digital Signal Processing I **Exam 3** **Fall 2017**

Session 40 **1 Dec. 2017**

Cover Sheet

Test Duration: 60 minutes.

Open Book but Closed Notes. One 8.5 x 11 crib sheet allowed

Calculators NOT allowed.

Show all work. More credit for approach than final answer.

This test contains **THREE** problems.

All work should be done on the blank pages provided.

Your answer to each part of the exam should be clearly labeled.

Problem 1. Determine the impulse response $h_a[n]$ in Figure 1(b) so that the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). You should obviously work in the frequency domain and show all work. Plot the magnitude of the DTFT of $h_a[n]$ over $-\pi < \omega < \pi$.

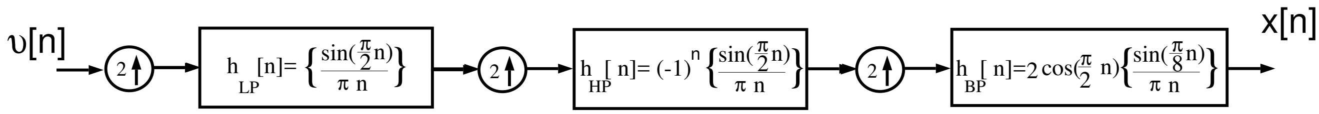

Figure 1(a).

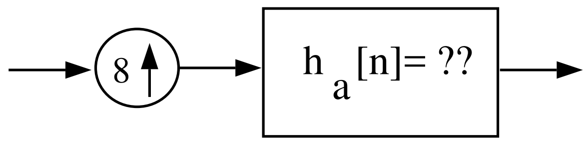

Figure 1(b).

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Problem 2. Consider a causal FIR filter of length $M = 4$ with impulse response as defined below:

$$h[n] = \cos\left(\frac{\pi}{4}n\right)\{u[n] - u[n-4]\}$$

Consider a DT sinewave $x[n]$ of length $N = 8$ as defined below:

$$x[n] = \sin\left(\frac{2\pi}{8}n\right)\{u[n] - u[n-8]\}$$

$y[n]$ is formed as the linear convolution of $x[n]$ with $h[n]$ as:

$$y[n] = x[n] * h[n]$$

We then take the last $M - 1 = 3$ values of $y[n]$ and time-domain alias them into the first three values to form a sequence of length 8, denoted $y_a[n]$, according to:

$$y_a[n] = y[n] + y[n+8], n = 0, 1, 2 \quad (1)$$
$$y_a[n] = y[n], n = 3, 4, 5, 6, 7 \quad (2)$$

Determine an expression for $y_a[n]$ similar to the expression for $x[n]$ above; list the 8 numerical values of $y_a[n], n = 0, 1, ..., 7$ in sequence form.

NOTE 1: the trigonometric values below are useful in this problem AND in Problem 3 as well:

$$\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} \qquad \sin\left(\frac{5\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} \quad (3)$$

Just carry any $\sqrt{2}$ factors along as scalars throughout your derivations. Show all work.

NOTE 2: Using concepts learned in class, this problem should only take a few lines, with the most work involving what the frequency response of the filter is doing at the frequency $\omega = \frac{2\pi}{8} = \frac{\pi}{4}$.

NOTE 3: There won't be a lot of points for brute force calculations that do not show what you learned in class for Exam 3.

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Problem 3. NOTE: the trigonometric values in Eqn (3) in Problem 2 will be useful in part (c) below as well; just carry any $\sqrt{2}$ factors along as a scalar. Consider a causal FIR filter of length $M = 10$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \left\{ \frac{\sin\left[\frac{\pi}{4}(n+\ell 10)\right]}{\pi(n+\ell 10)} + \frac{\sin\left[\frac{\pi}{2}(n+\ell 10)\right]}{\pi(n+\ell 10)} + \frac{\sin\left[\frac{3\pi}{4}(n+\ell 10)\right]}{\pi(n+\ell 10)} \right\} \{u[n] - u[n-10]\}$$

(a) Determine the 10-pt DFT of $h_p[n]$, denoted $H_{10}(k)$, for $0 \leq k \leq 9$. Write your answer in sequence form to indicate the numerical values of $H_{10}(k), k = 0, 1, ..., 9$.

(b) Consider the sequence $x[n]$ of length $L = 10$ below, equal to a sum of 10 finite-length sinewaves, each having a different amplitude as indicated below.

$$x[n] = \sum_{k=0}^{9}(10-k)e^{jk\frac{2\pi}{10}n}\{u[n] - u[n-10]\}$$

$y_{10}[n]$ is formed by computing $X_{10}(k)$ as a 10-pt DFT of $x[n]$, $H_{10}(k)$ as a 10-pt DFT of $h[n]$ and, finally, then $y_{10}[n]$ is computed as the 10-pt inverse DFT of $Y_{10}(k) = X_{10}(k)H_{10}(k)$. Express the result $y_{10}[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above. **Clearly indicate which of the 10 equi-spaced frequencies are nulled out, i.e., are not present in** $y_{10}[n]$.

Next, consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h_p[n] = 2\sqrt{2}\, j \sum_{\ell=-\infty}^{\infty} \left\{ \frac{\sin[\pi(n-1+\ell 8)]}{\pi(n-1+\ell 8)} - \frac{\sin[\pi(n+1+\ell 8)]}{\pi(n+1+\ell 8)} \right\} \{u[n] - u[n-8]\}$$

(c) Determine all 8 numerical values of the 8-pt DFT of $h_p[n]$, denoted $H_8(k)$, for $0 \leq k \leq 7$. Write your answer in sequence form to indicate the numerical values of $H_8(k), k = 0, 1, ..., 7$.

(d) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of 8 finite-length sinewaves as indicated below.

$$x[n] = \sum_{k=0}^{7} e^{jk\frac{2\pi}{8}n}\{u[n] - u[n-8]\}$$

$y_8[n]$ is formed by computing $X_8(k)$ as a 8-pt DFT of $x[n]$, $H_8(k)$ as a 8-pt DFT of $h[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above. **Clearly indicate which of the 8 equi-spaced frequencies are nulled out, i.e., are not present in** $y_8[n]$.

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