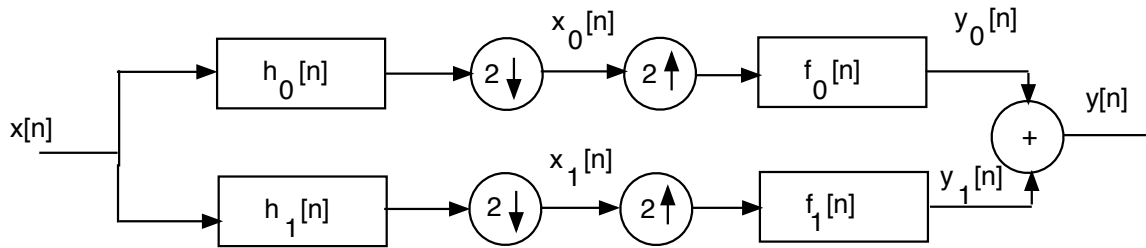


Problem 1. In the system below, the two analysis filters, $h_0[n]$ and $h_1[n]$, and the two synthesis filters, $f_0[n]$ and $f_1[n]$, form a Quadrature Mirror Filter (QMF). Specifically,

$$h_1[n] = (-1)^n h_0[n] \quad f_0[n] = h_0[n] \quad f_1[n] = -h_1[n]$$

The DTFT of the halfband filter $h_0[n]$ above may be expressed as follows:

$$H_0(\omega) = \begin{cases} 0, & -\pi < \omega < -\frac{3\pi}{4} \\ e^{j\frac{\omega}{2}} \sqrt{\frac{2}{\pi}} \sqrt{\frac{3\pi}{4} + \omega}, & -\frac{3\pi}{4} < \omega < -\frac{\pi}{4} \\ e^{j\frac{\omega}{2}}, & |\omega| < \frac{\pi}{4} \\ e^{j\frac{\omega}{2}} \sqrt{\frac{2}{\pi}} \sqrt{\frac{3\pi}{4} - \omega}, & \frac{\pi}{4} < \omega < \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < \omega < \pi \end{cases}$$



Determine mathematically (include as much detail as possible) if the lowpass half-band filter above satisfies the condition required for Perfect Reconstruction. Be sure to clearly state what that condition is (don't need to rederive it) and then show whether it is satisfied with the filter $h_0[n]$, showing as much detail as possible.

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Problem 2. For all parts of this problem, $x[n]$ is the fine-length sinewave of length $L = 8$ with frequency $\omega_o = \pi$ defined below, and $h[n]$ is a causal filter of length $M = 4$ which may be expressed in sequence form as $h[n] = \{1, -1, 1, -1\}$.

$$x[n] = e^{j\pi n} \{u[n] - u[n - 8]\} \qquad h[n] = (-1)^n \{u[n] - u[n - 4]\}$$

- Compute the linear convolution of $x[n]$ and $h[n]$. Indicate which points are the transient points (partial overlap) at the beginning and end, and also which points are “pure” sinewave (full overlap.)
- With $X_N(k)$ computed as the 8-pt DFT of $x[n]$ and $H_N(k)$ computed as the 8-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is formed. Determine the $N = 8$ values of the 8-pt Inverse DFT of $Y_N(k) = X_N(k)H_N(k)$.
- Using your answer to (a), explain your answer to (b) by mathematically illustrating the time-domain aliasing effect.
- The product sequence $Y_N(k) = X_N(k)H_N(k)$, formed as directly above with $N = 8$, is used in Eqn (1). Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$, computed according to Eqn (1) below:

$$Y_r(\omega) = \sum_{k=0}^{N-1} Y_N(k) \frac{\sin \left[\frac{N}{2} \left(\omega - \frac{2\pi k}{N} \right) \right]}{N \sin \left[\frac{1}{2} \left(\omega - \frac{2\pi k}{N} \right) \right]} e^{-j \frac{N-1}{2} \left(\omega - \frac{2\pi k}{N} \right)} \quad (1)$$

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Problem 3. Consider a causal FIR filter of length $M = 12$ with impulse response as defined below:

$$h[n] = \sum_{\ell=-\infty}^{\infty} \frac{\sin\left[\frac{\pi}{4}(n + \ell 12)\right]}{\pi(n + \ell 12)} \frac{\sin\left[\frac{3\pi}{4}(n + \ell 12)\right]}{\pi(n + \ell 12)} \{u[n] - u[n - 12]\}$$

- (a) Determine all 12 numerical values of the 12-pt DFT of $h[n]$, denoted $H_{12}(k)$, for $0 \leq k \leq 11$. List the values clearly: $H_{12}(k) = ?$, for $0 \leq k \leq 11$.
- (b) Consider the sequence $x[n]$ of length $L = 12$ below, equal to a sum of 8 finite-length sinewaves.

$$x[n] = \sum_{k=0}^{11} e^{jk\frac{2\pi}{12}n} \{u[n] - u[n - 12]\}$$

$y_{12}[n]$ is formed by computing $X_{12}(k)$ as an 12-pt DFT of $x[n]$, $H_{12}(k)$ as an 12-pt DFT of $h[n]$, and then $y_{12}[n]$ as the 12-pt inverse DFT of $Y_{12}(k) = X_{12}(k)H_{12}(k)$. Express the result $y_{12}[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

- (c) Next, consider a causal signal of length $M = 12$ with impulse response as defined below:

$$x[n] = \sum_{\ell=-\infty}^{\infty} 8 \left\{ \frac{\sin\left[\frac{\pi}{4}(n + \ell 12)\right]}{\pi(n + \ell 12)} \right\}^2 \cos\left(\frac{\pi}{2}(n + \ell 12)\right) \{u[n] - u[n - 12]\}$$

Determine all 12 numerical values of the 12-pt DFT of $x[n]$, denoted $X_{12}(k)$, for $0 \leq k \leq 11$. List all 12 numerical values clearly.

- (d) **EXTRA CREDIT.** Next, consider a causal signal of length $M = 16$ with impulse response as defined below:

$$x[n] = \sum_{\ell=-\infty}^{\infty} 16j \left\{ \frac{\sin\left[\frac{\pi}{8}(n + \ell 16)\right]}{\pi(n + \ell 16)} \right\} \left\{ \frac{\sin\left[\frac{3\pi}{8}(n + \ell 16)\right]}{\pi(n + \ell 16)} \right\} \sin\left(\frac{\pi}{2}(n + \ell 16)\right) \{u[n] - u[n - 16]\}$$

Determine all 16 numerical values of the 16-pt DFT of $x[n]$, denoted $X_{16}(k)$, for $0 \leq k \leq 15$. List all 16 numerical values clearly.

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