NAME: 2013

Digital Signal Processing I **Exam 3** **Fall 2013**

Session 41 **25 Nov. 2013**

Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes. One 8.5 x 11 crib sheet allowed
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on the blank pages provided.
Your answer to each part of the exam should be clearly labeled.

Problem 1. [50 pts] For all parts of this problem, let $x[n]$ be a discrete-time rectangular pulse of length $L = 8$ and $h[n]$ be a discrete-time rectangular pulse of length $M = 4$ as defined below:

$$x[n] = u[n] - u[n-8] \qquad\qquad h[n] = u[n] - u[n-4]$$

Also, for all parts of this problem, the reconstructed spectrum is computed according to Equation 1 below:

$$Y_r(\omega) = \sum_{k=0}^{N-1} Y_N(k) \frac{\sin\left[\frac{N}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]}{N \sin\left[\frac{1}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]} e^{-j\frac{N-1}{2}\left(\omega - \frac{2\pi k}{N}\right)} \tag{1}$$

(a)-(i) With $X_N(k)$ computed as the 16-pt DFT of $x[n]$ and $H_N(k)$ computed as the 16-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is formed. Determine the $N = 16$ values of the 16-pt Inverse DFT of $Y_N(k) = X_N(k)H_N(k)$.

(a)-(ii) The product sequence, $Y_N(k) = X_N(k)H_N(k)$, formed as directly above with $N = 16$, is used in Eqn (1). Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(b)-(i) With $X_N(k)$ computed as the 12-pt DFT of $x[n]$ and $H_N(k)$ computed as the 12-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is formed. Determine the $N = 12$ values of the 12-pt Inverse DFT of $Y_N(k) = X_N(k)H_N(k)$.

(b)-(ii) The product sequence $Y_N(k) = X_N(k)H_N(k)$, formed as directly above with $N = 12$, is used in Eqn (1). Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(c)-(i) With $X_N(k)$ computed as the 8-pt DFT of $x[n]$ and $H_N(k)$ computed as the 8-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is formed. Determine the $N = 8$ values of the 8-pt Inverse DFT of $Y_N(k) = X_N(k)H_N(k)$.

(c)-(ii) The product sequence $Y_N(k) = X_N(k)H_N(k)$, formed as directly above with $N = 8$, is used in Eqn (1). Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

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Problem 2. [50 points] Consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \frac{\sin\left[\frac{5\pi}{8}(n+\ell 8)\right]}{\pi(n+\ell 8)} \{u[n] - u[n-8]\}$$

(a) Determine all 8 numerical values of the 8-pt DFT of $h_p[n]$, denoted $H_8(k)$, for $0 \le k \le 7$. List the values clearly: $H_8(0) =?$, $H_8(1) =?$, $H_8(2) =?$, $H_8(3) =?$, $H_8(4) =?$, $H_8(5) =?$, $H_8(6) =?$, $H_8(7) =?$.

(b) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of 8 finite-length sinewaves.

$$x[n] = \sum_{k=0}^{7} e^{jk\frac{2\pi}{8}n} \{u[n] - u[n-8]\}$$

$y_8[n]$ is formed by computing $X_8(k)$ as an 8-pt DFT of $x[n]$, $H_8(k)$ as an 8-pt DFT of $h_p[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

Next, consider a causal FIR filter of length $M = 8$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \frac{8}{3} \frac{\sin\left[\frac{5\pi}{8}(n+\ell 8)\right]}{\pi(n+\ell 8)} \frac{\sin\left[\frac{3\pi}{8}(n+\ell 8)\right]}{\pi(n+\ell 8)} \{u[n] - u[n-8]\}$$

(c) Determine all 8 numerical values of the 8-pt DFT of $h_p[n]$, denoted $H_8(k)$, for $0 \le k \le 7$. List the values clearly: $H_8(0) =?$, $H_8(1) =?$, $H_8(2) =?$, $H_8(3) =?$, $H_8(4) =?$, $H_8(5) =?$, $H_8(6) =?$, $H_8(7) =?$.

(d) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of 8 finite-length sinewaves.

$$x[n] = \sum_{k=0}^{7} e^{jk\frac{2\pi}{8}n} \{u[n] - u[n-8]\}$$

$y_8[n]$ is formed by computing $X_8(k)$ as an 8-pt DFT of $x[n]$, $H_8(k)$ as an 8-pt DFT of $h_p[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

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