NAME: **30 Nov. 2011**
Digital Signal Processing I **Exam 3** **Fall 2011**
Session 40 **30 Nov. 2011**

Cover Sheet

WRITE YOUR NAME ON EACH EXAM SHEET

Test Duration: 60 minutes.

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **two** problems.

All work should be done in the space provided.

Do **not** just write answers; provide concise reasoning for each answer.

Problem 1. Let $x[n]$ be a discrete-time rectangular pulse of length $L = 5$ and $h[n]$ be a discrete-time rectangular pulse of length $M = 3$ as defined below:

$$x[n] = u[n] - u[n-5] \qquad\qquad h[n] = u[n] - u[n-3]$$

(a) With $X_N(k)$ computed as the 5-pt DFT of $x[n] = u[n] - u[n-5]$ and $H_N(k)$ computed as the 5-pt DFT of $h[n] = u[n] - u[n-3]$. The 5-point sequence $y_5[n]$ is computed as the 5-pt inverse DFT of the product $Y_N(k) = X_N(k)H_N(k)$. Write out the 5 numerical values of $y_5[n]$ in sequence form as $\{y_5[0], y_5[1], y_5[2], y_5[3], y_5[4]\}$.

(b) With $X_N(k)$ computed as the 8-pt DFT of $x[n] = u[n] - u[n-5]$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n-3]$. The 8-point sequence $y_8[n]$ is computed as the 8-pt inverse DFT of the product $Y_N(k) = X_N(k)H_N(k)$. Write out the 8 numerical values of $y_8[n]$ in sequence form.

(c) With $X_N(k)$ computed as the 10-pt DFT of $x[n] = u[n] - u[n-5]$ and $H_N(k)$ computed as the 10-pt DFT of $h[n] = u[n] - u[n-3]$. The 10-point sequence $y_{10}[n]$ is computed as the 10-pt inverse DFT of the product $Y_N(k) = X_N(k)H_N(k)$. Write out the 10 numerical values of $y_{10}[n]$ in sequence form.

Problem 2.

For all parts of this problem, the reconstructed spectrum is computed according to the equation below:

$$Y_r(\omega) = \sum_{k=0}^{N-1} Y_N(k) \frac{\sin\left[\frac{N}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]}{N \sin\left[\frac{1}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]} e^{-j\frac{N-1}{2}\left(\omega - \frac{2\pi k}{N}\right)} \tag{1}$$

Let $x[n]$ be a finite-length sinewave of length $L = 8$ and $h[n]$ be a discrete-time rectangular pulse of length $M = 5$ as defined below:

$$x[n] = e^{j\frac{\pi}{2}n}\left\{u[n] - u[n-8]\right\} \qquad\qquad h[n] = u[n] - u[n-5]$$

(a) With $X_N(k)$ computed as the 16-pt DFT of $x[n]$ and $H_N(k)$ computed as the 16-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 16$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(b) With $X_N(k)$ computed as the 12-pt DFT of $x[n] = e^{j\frac{\pi}{2}n}\{u[n] - u[n-8]\}$ and $H_N(k)$ computed as the 12-pt DFT of $h[n] = u[n]-u[n-5]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 12$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(c) The answer to this part will be useful in determining the answer to part (d). $X_N(k)$ computed as the 8-pt DFT of $x[n] = e^{j\frac{\pi}{2}n}\{u[n] - u[n-8]\}$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n-5]$. Develop and delineate your answers to each of the four steps below in the space below. Simplify each answer as much as possible.

(i) Determine a closed-form expression for the 8-pt DFT, $X_N(k)$, of $x[n] = e^{j\frac{\pi}{2}n}\{u[n] - u[n-8]\}$.

(ii) Determine a closed-form expression for the 8-pt DFT, $H_N(k)$, of $h[n] = \{u[n] - u[n-5]\}$.

(iii) Determine a closed-form expression for the product $Y_N(k) = X_N(k)H_N(k)$.

(iv) Determine a simple, closed-form expression for $y_8[n]$ equal to the 8-pt inverse DFT of $Y_N(k) = X_N(k)H_N(k)$. Note that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.

(d) With $X_N(k)$ computed as the 8-pt DFT of $x[n] = e^{j\frac{\pi}{2}n}\{u[n] - u[n-8]\}$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n-5]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 8$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$. Note that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.