WRITE YOUR NAME ON EACH EXAM SHEET

Test Duration: 60 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains two problems.
All work should be done in the space provided.
Do not just write answers; provide concise reasoning for each answer.
Problem 1. Let \( x[n] \) be a discrete-time rectangular pulse of length \( L = 5 \) and \( h[n] \) be a discrete-time rectangular pulse of length \( M = 3 \) as defined below:

\[
x[n] = u[n] - u[n - 5] \\
h[n] = u[n] - u[n - 3]
\]

(a) With \( X_N(k) \) computed as the 5-pt DFT of \( x[n] = u[n] - u[n - 5] \) and \( H_N(k) \) computed as the 5-pt DFT of \( h[n] = u[n] - u[n - 3] \). The 5-point sequence \( y_5[n] \) is computed as the 5-pt inverse DFT of the product \( Y_N(k) = X_N(k)H_N(k) \). Write out the 5 numerical values of \( y_5[n] \) in sequence form as \( \{y_5[0], y_5[1], y_5[2], y_5[3], y_5[4]\} \).
(b) With $X_N(k)$ computed as the 8-pt DFT of $x[n] = u[n] - u[n - 5]$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n - 3]$. The 8-point sequence $y_8[n]$ is computed as the 8-pt inverse DFT of the product $Y_N(k) = X_N(k)H_N(k)$. Write out the 8 numerical values of $y_8[n]$ in sequence form.
(c) With $X_N(k)$ computed as the 10-pt DFT of $x[n] = u[n] - u[n-5]$ and $H_N(k)$ computed as the 10-pt DFT of $h[n] = u[n] - u[n-3]$. The 10-point sequence $y_{10}[n]$ is computed as the 10-pt inverse DFT of the product $Y_N(k) = X_N(k)H_N(k)$. Write out the 10 numerical values of $y_{10}[n]$ in sequence form.
Problem 2.

For all parts of this problem, the reconstructed spectrum is computed according to the equation below:

\[ Y_r(\omega) = \sum_{k=0}^{N-1} Y_N(k) \frac{\sin \left( \frac{N}{2} \left( \omega - \frac{2\pi k}{N} \right) \right)}{N \sin \left( \frac{1}{2} \left( \omega - \frac{2\pi k}{N} \right) \right)} e^{-j\frac{N-1}{2} \left( \omega - \frac{2\pi k}{N} \right)} \]  

(1)

Let \( x[n] \) be a finite-length sinewave of length \( L = 8 \) and \( h[n] \) be a discrete-time rectangular pulse of length \( M = 5 \) as defined below:

\[ x[n] = e^{j\frac{\pi}{4} n} \{ u[n] - u[n-8] \} \quad h[n] = u[n] - u[n-5] \]

(a) With \( X_N(k) \) computed as the 16-pt DFT of \( x[n] \) and \( H_N(k) \) computed as the 16-pt DFT of \( h[n] \), the product \( Y_N(k) = X_N(k)H_N(k) \) is used in Eqn (1) with \( N = 16 \). Write a closed-form expression for the reconstructed spectrum \( Y_r(\omega) \).
(b) With $X_N(k)$ computed as the 12-pt DFT of $x[n] = e^{j \frac{\pi}{2} n} \{ u[n] - u[n - 8] \}$ and $H_N(k)$ computed as the 12-pt DFT of $h[n] = u[n] - u[n - 5]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 12$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$. 


(c) The answer to this part will be useful in determining the answer to part (d). $X_N(k)$ computed as the 8-pt DFT of $x[n] = e^{j \frac{\pi}{4} n} \{u[n] - u[n - 8]\}$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n - 5]$. Develop and delineate your answers to each of the four steps below in the space below. Simplify each answer as much as possible.

(i) Determine a closed-form expression for the 8-pt DFT, $X_N(k)$, of $x[n] = e^{j \frac{\pi}{4} n} \{u[n] - u[n - 8]\}$.
(ii) Determine a closed-form expression for the 8-pt DFT, $H_N(k)$, of $h[n] = \{u[n] - u[n - 5]\}$.
(iii) Determine a closed-form expression for the product $Y_N(k) = X_N(k)H_N(k)$.
(iv) Determine a simple, closed-form expression for $y_8[n]$ equal to the 8-pt inverse DFT of $Y_N(k) = X_N(k)H_N(k)$. Note that $\sin \left( \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$ and $\sin \left( \frac{5\pi}{4} \right) = -\frac{1}{\sqrt{2}}$. 


(d) With $X_N(k)$ computed as the 8-pt DFT of $x[n] = e^{j\frac{\pi}{8}n} \{u[n] - u[n-8]\}$ and $H_N(k)$ computed as the 8-pt DFT of $h[n] = u[n] - u[n-5]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 8$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$. Note that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$. 