ECE 538

Exam 3 Solution — Fall 2010

Problem 1:

Since $y[n] = x[n] * h[n]$

$L=5 \quad M=3$

is of length $M+L-1 = 5+3-1 = 7$

(i) for $N=12 \Rightarrow 12 > 7 \Rightarrow$ no time-domain aliasing

$$Y_r(\omega) = Y(\omega) = X(\omega)H(\omega)$$

$$= \underbrace{\frac{\sin\left(\frac{5}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j2\omega} \frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\omega}}$$

(ii) $N = 8 > 7 \Rightarrow Y_r(\omega) = Y(\omega)$

(iii) $N = 5 < 7 \Rightarrow$ time-domain aliasing

$$y_r[n] = y[n] + y[n+5]$$

$7-5=2$ points at end aliased into 2 points at the beginning

$$y[n] = x[n] * h[n] = \{\underset{\uparrow}{1}, 2, 3, 3, 3, 2, 1\}$$

$$y[n+5] = \{1, 2, 3, 3, 3, 2, 1\}$$

$y_R[n] = \{3, 3, 3, 3, 3\}$ of length 5

Prob. 1 Soln (cont.)

answer to (iii):

$$Y_r(\omega) = 3\,\frac{\sin\left(\frac{5}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}\,e^{-j2\omega}$$

Prob. 1 (b). For sine-window showed in class

$$Y(\omega) = e^{-j\frac{7}{2}\omega}\,\frac{1}{2}\left\{\frac{\sin\left(\frac{8}{2}\left(\omega-\frac{\pi}{8}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega-\frac{\pi}{8}\right)\right)} + \frac{\sin\left(\frac{8}{2}\left(\omega+\frac{\pi}{8}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega+\frac{\pi}{8}\right)\right)}\right\}$$

Zero-crossings at in $(0,\pi)$: $\frac{3\pi}{8}$, $\underbrace{\frac{3\pi}{8}+\frac{2\pi}{8}}_{\frac{5\pi}{8}}$, $\underbrace{\frac{3\pi}{8}+\frac{4\pi}{8}}_{\frac{7\pi}{8}}$

in $(\pi, 2\pi)$: $\underbrace{\frac{3\pi}{8}+\frac{6\pi}{8}}_{\frac{9\pi}{8}}$, $\underbrace{\frac{3\pi}{8}+\frac{8\pi}{8}}_{\frac{11\pi}{8}}$, $\underbrace{\frac{3\pi}{8}+\frac{10\pi}{8}}_{\frac{13\pi}{8}}$

(b)-(i) $Y_N(3) = Y_r(\omega)\Big|_{\omega = 3\frac{2\pi}{16} = \frac{3\pi}{8}}$ ⇒ Zero crossing

$Y_r(\omega) = Y(\omega)$ since $N = 16 > L = 8$

$$\boxed{Y_N(3) = 0}$$

(b)-(ii) $Y_N(5) = Y(\omega)\Big|_{\omega = 5\frac{2\pi}{16} = \frac{5\pi}{8}}$ ⇒ Zero crossing

$$\boxed{Y_N(5) = 0}$$

(b) - (iii): $Y_N(0) = Y(\omega)\Big|_{\omega=0}$

$$Y(0) = \frac{1}{2}\left\{\frac{\sin\left(-\frac{\pi}{2}\right)}{\sin\left(-\frac{\pi}{16}\right)} + \frac{\sin\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{16}\right)}\right\}$$

$$= \frac{1}{2}\ 2\ \frac{1}{\sin\left(\frac{\pi}{16}\right)} \approx \frac{16}{\pi}$$

Prob. 2

(a) $H(\omega) = 1 + 2e^{-j\omega} + e^{-j2\omega}$

$$= \left\{\frac{\sin\left(\frac{2}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\frac{\omega}{2}}\right\}^2$$

$$= \frac{\sin^2(\omega)}{\sin^2\left(\frac{\omega}{2}\right)} e^{-j\omega}$$

$$H_8(k) = H(\omega)\Big|_{\omega = \frac{2\pi}{8}k} = k\frac{\pi}{4}$$

$$H_8(k) = \left\{\frac{\sin\left(k\frac{\pi}{4}\right)}{\sin\left(k\frac{\pi}{8}\right)}\right\}^2 e^{-jk\frac{\pi}{4}}$$

eight $\quad k = 0, 1, \ldots, 7$

(b) Linear processing ⇒ do each sinewave individually and then sum

⇒ each sinewave is the special case

i) $\omega = \frac{\pi}{4} = (1)\frac{2\pi}{8} \Rightarrow k = 1$

$$H_8(1) = \left\{ \frac{\sin\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{8}\right)} \right\}^2 e^{-j\frac{\pi}{4}}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{2}}\right)} e^{-j\frac{\pi}{4}}$$

$$= \frac{1}{1 - .707} e^{-j\frac{\pi}{4}}$$

$$= 3.414\, e^{-j\pi/4}$$

$\sin^2\theta$

$= \frac{1}{2} - \frac{1}{2}\cos(2\theta)$

$\sin^2\left(\frac{\pi}{8}\right)$

$= \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{2}}$

$\Rightarrow$ output term

$$2\,(3.414)\cos\left(\frac{\pi}{4}n - \frac{\pi}{4}\right)\left\{u[n] - u[n-8]\right\}$$

(ii) $\omega = \frac{\pi}{2} = 2\,\frac{2\pi}{8} \Rightarrow k = 2$

$$H_8(2) = \left\{ \frac{\sin\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{4}\right)} \right\}^2 e^{-j\frac{\pi}{2}} = 2\,e^{-j\frac{\pi}{2}}$$

$\Rightarrow$ output term:

$$3\,(2)\cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)\left\{u[n] - u[n-8]\right\}$$

(iii) $\omega = \pi = (4)\frac{2\pi}{8} \Rightarrow k = 4$

$$H_8(4) = \left\{\frac{\sin(\pi)}{\sin\left(\frac{\pi}{2}\right)}\right\}^2 e^{-j\pi} = 0$$

no contribution to output

Answer:

$$y_8[n] = \left\{\left\{6.82 \cos\left(\frac{\pi}{4}(n-1)\right) + 6 \cos\left(\frac{\pi}{2}(n-1)\right)\right\} \cdot \{u[n] - u[n-8]\}\right\}$$