Digital Signal Processing I **Exam 3** **Fall 2010**
Session 41 **3 Dec. 2010**

Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on blank 8.5” x 11” white sheets of paper.
Do **not** return this test sheet, just return your answer sheets.

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Problem 1. [65 pts]

For all parts of this problem, the reconstructed spectrum is computed according to Equation 1 below:

$$Y_r(\omega) = \sum_{k=0}^{N-1} Y_N(k) \frac{\sin\left[\frac{N}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]}{N \sin\left[\frac{1}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]} e^{-j\frac{N-1}{2}\left(\omega - \frac{2\pi k}{N}\right)} \qquad (1)$$

(a) Let $x[n]$ be a discrete-time rectangular pulse of length $L = 5$ and $h[n]$ be a discrete-time rectangular pulse of length $M = 3$ as defined below:

$$x[n] = u[n] - u[n-5] \qquad\qquad h[n] = u[n] - u[n-3]$$

(i) With $X_N(k)$ computed as the 12-pt DFT of $x[n]$ and $H_N(k)$ computed as the 12-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 12$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(ii) With $X_N(k)$ computed as the 8-pt DFT of $x[n]$ and $H_N(k)$ computed as the 8-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 8$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(iii) With $X_N(k)$ computed as the 5-pt DFT of $x[n]$ and $H_N(k)$ computed as the 5-pt DFT of $h[n]$, the product $Y_N(k) = X_N(k)H_N(k)$ is used in Eqn (1) with $N = 5$. Write a closed-form expression for the reconstructed spectrum $Y_r(\omega)$.

(b) Let $y[n]$ be a sine-window of length $L = 8$ as defined below. For all sub-parts of part (b), $Y_N(k)$ is computed as a 16-pt DFT of $y[n]$ and used in Eqn (1) with $N = 16$.

$$y[n] = \sin\left(\frac{\pi}{8}(n + 0.5)\right)\{u[n] - u[n-8]\}$$

(i) Write a closed-form expression for the resulting reconstructed spectrum $Y_r(\omega)$.

(ii) What is the numerical value of $Y_N(3)$? That is, what is the numerical value of the 16-pt DFT of $y[n]$ for the value $k = 3$? (Note that $y[n]$ is of length $L = 8$.)

(iii) What is the numerical value of $Y_N(5)$? That is, what is the numerical value of the 16-pt DFT of $y[n]$ for the value $k = 5$?

(iv) What is the numerical value of $Y_N(0)$? That is, what is the numerical value of the 16-pt DFT of $y[n]$ for the value $k = 0$? You can approximate $\sin\left(\frac{\pi}{16}\right) \approx \frac{\pi}{16}$.

PROBLEM 2 IS ON THE NEXT PAGE

Problem 2. [35 points] Consider a causal FIR filter of length $M = 3$ with impulse response starting at $n = 0$:

$$h[n] = \{1, 2, 1\}$$

Note that it may be helpful to realize that $h[n] = \{1, 1\} * \{1, 1\}$.

(a) Provide a closed-form expression for the 8-pt DFT of $h[n]$, denoted $H_8(k)$, as a function of k. Simplify as much as possible.

(b) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of several finite-length sinewaves.

$$x[n] = \left[2\cos\left(\frac{\pi}{4}n\right) + 3\cos\left(\frac{\pi}{2}n\right) + 4\cos(\pi n)\right]\{u[n] - u[n-8]\}$$

$y_8[n]$ is formed by computing $X_8(k)$ as an 8-pt DFT of $x[n]$, $H_8(k)$ as an 8-pt DFT of $h[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

Note: These may be useful to you: $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, and $\cos\left(\frac{\pi}{2}\right) = 0$.