Digital Signal Processing I **Exam 3** **Fall 2009**
Session 41 **4 Dec. 2009**

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems.
All work should be done on blank 8.5" x 11" white sheets of paper (NOT provided).
Do **not** return this test sheet, just return your answer sheets.

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Problem 1. [65 pts]
For all parts of this problem, the reconstructed spectrum is computed according to Equation 1 below:

$$X_r(\omega) = \sum_{k=0}^{N-1} X_N(k) \frac{\sin\left[\frac{N}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]}{N \sin\left[\frac{1}{2}\left(\omega - \frac{2\pi k}{N}\right)\right]} e^{-j\frac{N-1}{2}\left(\omega - \frac{2\pi k}{N}\right)} \qquad (1)$$

(a) Let $x[n]$ be a discrete-time rectangular pulse of length $L = 12$ as defined below:

$$x[n] = u[n] - u[n-12]$$

(i) $X_N(k)$ is computed as a 16-point DFT of $x[n]$ and used in Eqn (1) with $N = 16$. Write a closed-form expression for the resulting reconstructed spectrum $X_r(\omega)$.

(ii) $X_N(k)$ is computed as a 12-point DFT of $x[n]$ and used in Eqn (1) with $N = 12$. Write a closed-form expression for the resulting reconstructed spectrum $X_r(\omega)$.

(iii) $X_N(k)$ is computed as an 8-point DFT of $x[n]$ and used in Eqn (1) with $N = 8$. That is, $X_N(k)$ is obtained by sampling the DTFT of $x[n]$ at 8 equi-spaced frequencies between 0 and 2π. Write a closed-form expression for the resulting reconstructed spectrum $X_r(\omega)$.

(b) Let $x[n]$ be a discrete-time sinewave of length $L = 16$ as defined below. For all subparts of part (b), $X_N(k)$ is computed as a 16-pt DFT of $x[n]$ and used in Eqn (1) with $N = 16$.

$$x[n] = \cos\left(\frac{\pi}{4}n\right)\{u[n] - u[n-16]\}$$

(i) Write a closed-form expression for the resulting reconstructed spectrum $X_r(\omega)$.

(ii) What is the numerical value of $X_r(\frac{\pi}{8})$? The answer is a number and you do not need a calculator to determine the value; this also applies to the next 2 parts.

(iii) What is the numerical value of $X_r(\frac{\pi}{4})$?

(iv) What is the numerical value of $X_r(\frac{7\pi}{4})$?

Problem 2. [35 points] Consider a causal FIR filter of length $M = 3$ with impulse response

$$h[n] = \{1, -2, 1\}$$

(a) Provide a closed-form expression for the 8-pt DFT of $h[n]$, denoted $H_8(k)$, as a function of k. Simplify as much as possible.

(b) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of several finite-length sinewaves.

$$x[n] = \left[3 + \cos\left(\frac{\pi}{2}n\right) + 2\cos(\pi n)\right]\{u[n] - u[n-8]\}$$

$y_8[n]$ is formed by computing $X_8(k)$ as an 8-pt DFT of $x[n]$, $H_8(k)$ as an 8-pt DFT of $h[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.