Digital Signal Processing I **Exam 3** **Fall 2008**
Session 38 **24 Nov. 2007**

Cover Sheet

Test Duration: 55 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done on blank 8.5" x 11" white sheets of paper (NOT provided).
Do **not** return this test sheet, just return your answer sheets.

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Problem 1. [30 pts]

Let $x[n]$ be of length $L = 7$, i.e., $x[n] = 0$ for $n < 0$ and $n \geq 7$, and $h[n]$ also be of length $M = 7$. Let $X_8(k)$ and $H_8(k)$ denote 8-point DFT's of $x[n]$ and h[n], respectively. The 8-point inverse DFT of the product $Y_8(k) = X_8(k)H_8(k)$, denoted $y_8[n]$, produces the following values:

n	0	1	2	3	4	5	6	7
$y_8[n]$	-2	0	-2	0	-2	0	7	0

Let $X_{10}(k)$ and $H_{10}(k)$ denote the 10-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 10-point inverse DFT of the product $Y_{10}(k) = X_{10}(k)H_{10}(k)$, denoted $y_{10}[n]$, produces the following values:

n	0	1	2	3	4	5	6	7	8	9
$y_{10}[n]$	-2	0	-2	0	-1	0	7	0	-1	0

Given $y_8[n]$ and $y_{10}[n]$, find the **linear** convolution of $x[n]$ and $h[n]$, i. e., list all of the numerical values of $y[n] = x[n] * h[n]$.

Problem 2. [30 points]

A signal $x[n]$ of length 12 is broken up into two nonoverlapping blocks of length 6, denoted $x_1[n]$ and $x_2[n]$, respectively, for the purposes of filtering with $h[n] = \{-1, 2, -1\}$ via the overlap-add method. Specifically, $x[n] = x_1[n] + x_2[n-6]$, where

$$x_1[n] = \{1, 1, 1, 1, 1, 1\}$$

and

$$x_2[n] = \{-1, -1, -1, -1, -1, -1\}$$

(a) $y_1[n]$ is formed by computing $X_1(k)$ as an 8-pt DFT of $x_1[n]$, $H(k)$ as an 8-pt DFT of $h[n]$, and then $y_1[n]$ as the 8-pt inverse DFT of $Y_1(k) = X_1(k)H(k)$. Write out the values of $y_1[n]$ in sequence form (similar to how $x_1[n]$ and $x_2[n]$ are written out above.)

(b) $y_2[n]$ is formed by computing $X_2(k)$ as an 8-pt DFT of $x_2[n]$, $H(k)$ as an 8-pt DFT of $h[n]$, and then $y_2[n]$ as the 8-pt inverse DFT of $Y_2(k) = X_2(k)H(k)$. Write out the 8 values of $y_2[n]$ in sequence form.

(c) Show how $y_1[n]$ and $y_2[n]$ are combined to form the full linear convolution $y[n] = x[n] * h[n]$, via the overlap-add method.

(d) To reduce computation, consider FFT based processing of two blocks simultaneously. To this end, we form the complex-valued sequence

$$v[n] = x_1[n] + jx_2[n]$$

$V(k)$ is computed as an 8-pt DFT of $v[n]$. After that, $z[n]$ is computed as the 8-pt inverse DFT of the product $Z(k) = V(k)H(k)$, where $H(k)$ is the 8 pt-DFT of $h[n] = \{-1, 2, -1\}$. (**NOTE:** You do NOT have to compute any 8-pt DFTs to answer the two questions on the top of the next page.)

(i) Is the real part of $z[n]$ equal to the $y_1[n]$ found in part (a)? You must briefly justify/explain your answer.

(ii) Is the imaginary part of $z[n]$ equal to the $y_2[n]$ found in part (b)? You must briefly justify/explain your answer.

Problem 3. [30 points] Consider a causal FIR filter of length $M = 6$ with impulse response

$$h[n] = \{1, 1, 1, 1, 1, 1\}$$

(a) Provide a closed-form expression for the 8-pt DFT of $h[n]$, denoted $H_8(k)$, as a function of k. Simplify as much as possible.

(b) Consider the sequence $x[n]$ of length $L = 8$ below, equal to a sum of several finite-length sinewaves.

$$x[n] = \cos\left(\frac{\pi}{2}n\right) + 2\cos(\pi n), \quad n = 0, 1, ..., 7.$$

$y_8[n]$ is formed by computing $X_8(k)$ as an 8-pt DFT of $x[n]$, $H_8(k)$ as an 8-pt DFT of $h[n]$, and then $y_8[n]$ as the 8-pt inverse DFT of $Y_8(k) = X_8(k)H_8(k)$. Express the result $y_8[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.