Digital Signal Processing I **Exam 3** **Fall 2007**
Session 41 **30 Nov. 2007**

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Digital Signal Processing I — Exam 3 — Fall 2007
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Problem 1. [30 pts]

Let $x[n]$ be of length $L = 4$, i.e., $x[n] = 0$ for $n < 0$ and $n \geq 4$ and $h[n]$ be of length $M = 5$, i.e., $h[n] = 0$ for $n < 0$ and $n \geq 5$. Let $X_6(k)$ and $H_6(k)$ denote 6-point DFT's of $x[n]$ and h[n], respectively. The 6-point inverse DFT of the product $Y_6(k) = X_6(k)H_6(k)$, denoted $y_6[n]$, produces the following values:

n	0	1	2	3	4	5
$y_6[n]$	3	3	3	4	4	3

Let $X_5(k)$ and $H_5(k)$ denote the 5-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 5-point inverse DFT of the product $Y_5(k) = X_5(k)H_5(k)$, denoted $y_5[n]$, produces the following values:

n	0	1	2	3	4
$y_5[n]$	4	4	4	4	4

Given $y_6[n]$ and $y_5[n]$, find the **linear** convolution of $x[n]$ and $h[n]$, i. e., list all of the numerical values of $y[n] = x[n] * h[n]$.

Problem 2. [30 points]

Consider the autoregressive AR(2) process generated via the difference equation

$$x[n] = x[n-1] - \frac{1}{2}x[n-2] + \nu[n]$$

where $\nu[n]$ is a stationary white noise process with variance $\sigma_w^2 = 5/2 = 2.5$. (The value of σ_w^2 was chosen so that the autocorrelation values requested in part (a) are whole numbers.)

(a) Determine the numerical values of $r_{xx}[0]$, $r_{xx}[1]$, $r_{xx}[2]$, where $r_{xx}[m]$ is the autocorrelation sequence $r_{xx}[m] = E\{x[n]x[n-m]\}$. Then determine the value of $r_{xx}[3]$.

(b) Determine a simple, closed-form expression for the spectral density for $x[n]$, $S_{xx}(\omega)$, which may be expressed as the DTFT of $r_{xx}[m]$:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}[m]e^{-jm\omega}$$

(c) Consider the first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the corresponding minimum mean-square error.

(d) Consider the third-order predictor

$$\hat{x}[n] = -a_3(1)x[n-1] - a_3(2)x[n-2] - a_3(3)x[n-3]$$

Determine the numerical values of the optimum predictor coefficients $a_3(1)$, $a_3(2)$, and $a_3(3)$ and the corresponding minimum mean-square error.

Problem 3. [30 points]

As part of the first stage in a radix 2 FFT, a sequence $x[n]$ of length $N = 8$ is decomposed into two sequences of length 4 as

$$f_0[n] = x[2n], \quad n = 0, 1, 2, 3 \qquad\qquad f_1[n] = x[2n+1], \quad n = 0, 1, 2, 3$$

We compute a 4 -pt. DFT of each of these two sequences as

$$f_0[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} F_0[k] \qquad\qquad f_1[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} F_1[k]$$

The specific values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, obtained from the length $N = 8$ sequence in question are listed in the Table below.

k	0	1	2	3
$F_0[k]$	0	4	0	0
$F_1[k]$	0	$-\sqrt{2}(2+2j)$	0	0
W_8^k	1	$\frac{1}{\sqrt{2}}(1-j)$	-j	$-\frac{1}{\sqrt{2}}(1+j)$

(20 pts) From the values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, and the values of $W_8^k = e^{-j\frac{2\pi}{8}k}$, $k = 0, 1, 2, 3$, provided in the Table, determine the numerical values of the actual $N = 8$-pt. DFT of $x[n]$ denoted $X_8[k]$ for $k = 0, 1, 2, 3, 4, 5, 6, 7$.

(10 pts) The underlying length $N = 8$ sequence $x[n]$ may be expressed as

$$x[n] = \cos\left(2\pi\frac{k_1}{8}n\right) + j\sin\left(2\pi\frac{k_2}{8}n\right), \quad n = 0, 1, ..., 7.$$

where k_1 and k_2 are both integers between 0 and 7, that is, $k_i \in \{0, 1, 2, 3, 4, 5, 6, 7\}$, $i = 1, 2$. Given the values of $X_8[k]$ for $k = 0, 1, ..., 7$ determined in part (a), determine the numerical values of k_1 and k_2.