ECE 538 Exam 3 Sol'n. Fall 2006

Prob. 1 Sol'n.

(a)
$$x_q[n] = \sum_{l=-\infty}^{\infty} (.9)^{n-lq} \underbrace{\{u[n]-u[n-q]\}}_{\hat{u}[n-lq]}$$

$$= \sum_{l=-\infty}^{0} (.9)^n \left((.9)^q\right)^{-l} \{u[n]-u[n-q]\}$$

$$= (.9)^n \sum_{l=0}^{\infty} \left((.9)^q\right)^{l} \{u[n]-u[n-q]\}$$

$$= \left(\frac{1}{1-.9^q}\right)(.9)^n \{u[n]-u[n-q]\}$$

(b) just a scalar multiple $\Rightarrow$ still have same functional dependence on n

So $\tilde{x}_q[n] = \dfrac{x_q[n]}{x_q[0]}$ is the same as $x[n]$ over $n = 0, 1, 2, \ldots, 8$

With one pole in the signal, all the aliasing terms simply contribute a scalar multiple difference relative to the original signal BECAUSE each aliased term has the same "shape" but a different starting value

Prob. 1 (c) Sol'n.

The key observation here is to write

$(.9)^{|n-4|}$ as $(.9)^{4-n} u[-n+3]$

$+ (.9)^{n-4} u[n-4]$

That is:

$$(.9)^{|n-4|} = \underbrace{(.9)^4 \left\{\frac{1}{(.9)}\right\}^n u[-n+3]}_{\text{Term 1}}$$

$$+ \underbrace{(.9)^{-4} (.9)^n u[n-4]}_{\text{Term 2}}$$

answer to (d): ⇒ going right to answer to part (d), this signal has 2 poles and thus, the time-aliased version will <u>not</u> simply be a scalar multiple of the original signal (truncated)

Continuing with part (c), consider the aliasing due to each part/term separately, then sum the results

To make the problem easier, I am going to first ignore the shift to the right by 4, find all the aliasing, and then shift my answer to the right by 4

Prob. 1 (c) Sol'n. (cont.)

- Term 1 shifted to left by 4 $\left(\text{replace } n \text{ by } n+4\right)$:

$$\sum_{\ell=-\infty}^{\infty} (.9)^{n-\ell 9} u[n-\ell 9] \left\{ u[n] - u[n-9] \right\}$$

already did this in part (a)

$$\Rightarrow 1.6324 \, (.9)^{n} \left\{ u[n] - u[n-9] \right\}$$

☆ shifting to the right by 4:

$$\Rightarrow 1.6324 \, (.9)^{n-4} \left\{ u[n] - u[n-9] \right\}$$

- Term 2 shifted to left by 4 $\left(\text{replace } n \text{ by } n+4\right)$:

$$\sum_{\ell=-\infty}^{\infty} (.9)^{-(n-\ell 9)} u[(n-\ell 9)-1] \left\{ u[n] - u[n-9] \right\}$$

$$= (.9)^{-n} \sum_{\ell=1}^{\infty} \left((.9)^9\right)^{\ell} \left\{ u[n] - u[n-9] \right\}$$

only the right-shifted versions contribute over $n=0,1,\ldots,8$

$$= (.9)^{-n} \left\{ \frac{1}{1-.9^9} - 1 \right\} = (.9)^{-n} \left[1.6324 - 1 \right] = 0.6324 (.9)^{-n}$$

☆ shifting back to the right by 4, and summing with Term 1 contribution yields final answer

$$y_9[n] = 0.6324 \, (.9)^{-(n-4)} + 1.6324 \, (.9)^{n-4}$$

for $n = 0, 1, 2, \ldots, 8$

Prob. 2 Soln.

$$\left.\begin{array}{ll} x[n], & L=6 \\ h[n], & M=6 \end{array}\right\} \; y[n]=x[n]*h[n] \text{ of length } L+M-1=6+6-1=11$$

Thus, with 8-pt DFT's, $11-8=3$ pts. at the end of $y[n]$ will be aliased into the first 3 points.

$$y[n]=\{\underbrace{1,2,3}_{},4,5,6,5,4,\underbrace{3,2,1}_{}\}$$

$$y_8[n]=y_p[n]=\{1+3,\ 2+2,\ 3+1,\ 4,5,6,5,4\}$$

Answer: $=\{4,4,4,4,5,6,5,4\}$

Prob. 3 Sol'n:

$$X_8(k) = F_0(k) + W_8^k F_1(k)$$
$$X_8(k+4) = F_0(k) - W_8^k F_1(k)$$

$k=0$: $X_8(0)=0$ $\quad X_8(4)=0$

In addition, it's also easy to see that for $k=3$

$k=3$: $X_8(3)=0$ $\quad X_8(7)=0$

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Prob. 3 Sol'n cont.

$k=1$: first form product

$$W_8^1 F_1(1) = \frac{1}{\sqrt{2}}(1-j)\cdot\sqrt{2}\,2(1+j)$$

$$= 2\{1+1\} = 4$$

Thus:

$$X_8(1) = F_0(1) + W_8^1 F_1(1) = 4+4=8$$

$$X_8(5) = F_0(1) - W_8^1 F_1(1) = 4-4=0$$

and:

$k=2$ first form product

$$W_8^2 F_1(2) = -j\,(4j) = 4$$

$$X_8(2) = 4+4=8$$

$$X_8(6) = 4-4=0$$

THUS: $X_8(k) = \{0, 8, 8, 0, 0, 0, 0, 0\}$

$$= 8\delta(k-1) + 8\delta(k-2)$$

Since:

$$e^{j2\pi\frac{k_0}{N}}\{u[n]-u[n-N]\} \xleftrightarrow[8]{DFT} N\delta(k-k_0)$$

$\therefore$ The answers are: $k_1=1$, $k_2=2$

$$x[n] = \left\{e^{j\frac{2\pi}{8}n} + e^{j\frac{2\pi(2)}{8}n}\right\}\{u[n]-u[n-8]\}$$