Sol'n to Prob. 1 Exam 3

(a) $h[n] = \frac{1}{3}\delta(n) + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$

$$H(e^{j\omega}) = \frac{1}{3}\left\{1 + e^{-j\omega} + e^{-j2\omega}\right\}$$

$$= \frac{1}{3}e^{-j\omega}\left\{1 + 2\cos\omega\right\}$$

$$= \frac{1}{3}\sum_{n=0}^{2} e^{-j\omega n} = \frac{1}{3}\frac{1 - e^{-j\omega 3}}{1 - e^{-j\omega}}$$

$$= \frac{1}{3}\frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\omega}$$

See Table 4.6 on pg. 305

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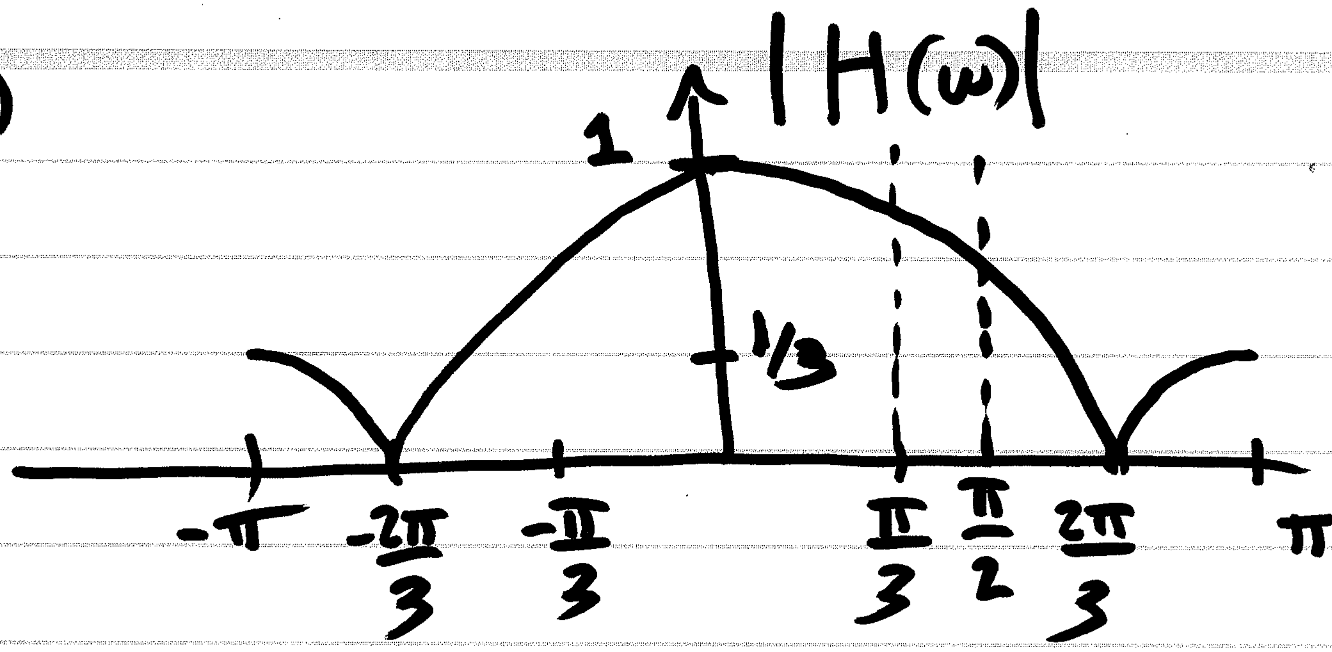
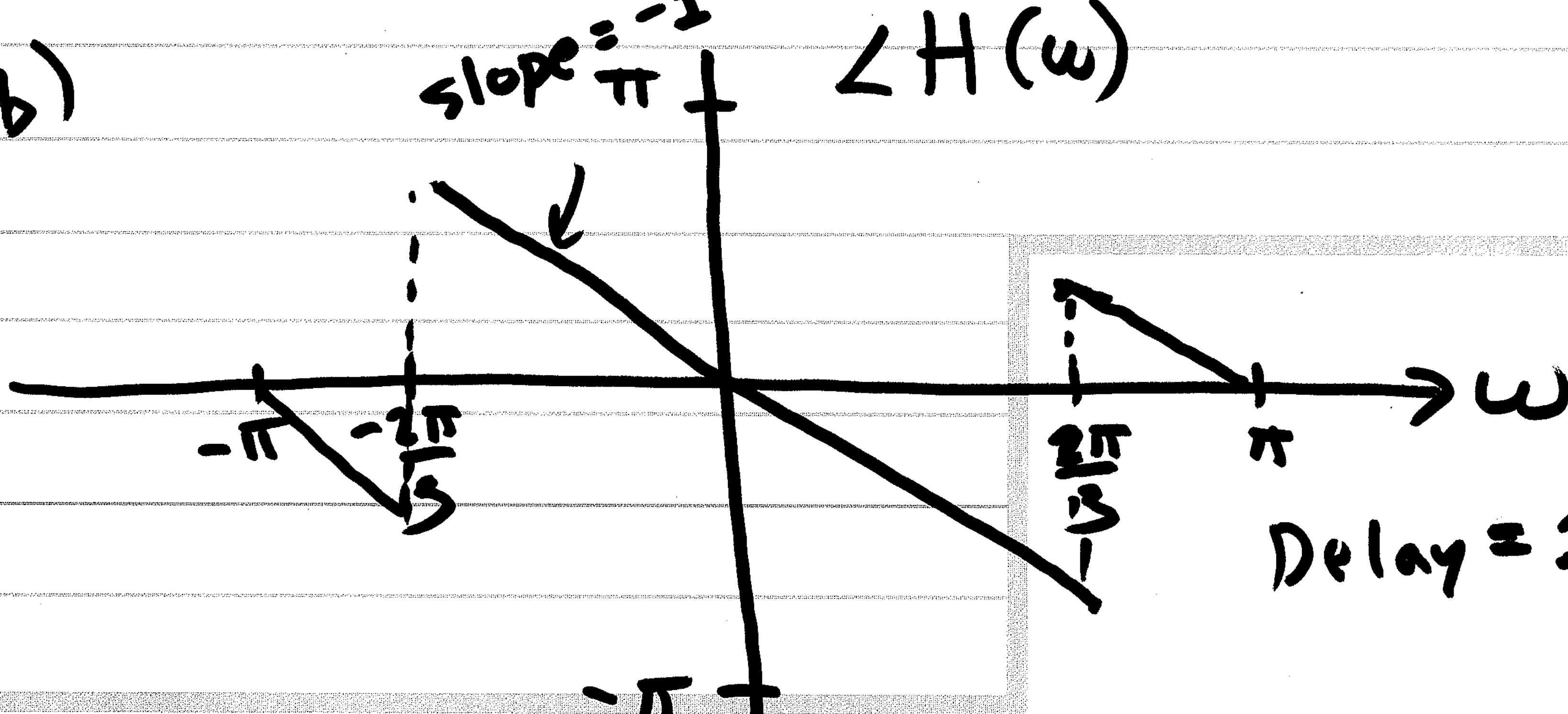
(a)



(b)



Delay = 1

(c) max dev. from 1 occurs at

$$\omega_p = \frac{\pi}{3} \Rightarrow \frac{1}{3}\,\frac{\sin\left(\frac{3}{2}\frac{\pi}{3}\right)}{\sin\left(\frac{1}{2}\frac{\pi}{3}\right)}$$

$$= \frac{1}{3}\,\frac{1}{1/2} = \frac{2}{3}$$

$$\delta_1 = 1 - \frac{2}{3} = \frac{1}{3}$$

(d) max dev from 0 in stopband occurs at ? check $\omega_s = \frac{\pi}{2}$ & $\omega = \pi$

at $\omega_s = \frac{\pi}{2}$: $\frac{1}{3}\,\frac{\sin\left(\frac{3}{2}\frac{\pi}{2}\right)}{\sin\left(\frac{1}{2}\frac{\pi}{2}\right)} = \frac{1}{3}\,\frac{1/\sqrt{2}}{1/\sqrt{2}} = \frac{1}{3}$

at $\omega = \pi$: $\frac{1}{3} \frac{\sin\left(\frac{3}{2}\pi\right)}{\sin\left(\frac{1}{2}\pi\right)} = \frac{1}{3} \frac{(-1)}{(1)}$

$$\left| H(\omega) \right| \Big|_{\omega=\pi} = \frac{1}{3}$$

$$\delta_2 = \frac{1}{3}$$

(e) $L = \frac{M-1}{2} \Big|_{M=3} = \frac{3-1}{2} = 1$

no. of extremal freqs $\Rightarrow L+2$ or $L+3$

$\Rightarrow$ either 3 or 4 $\Rightarrow$ ans $= 3$

$\Rightarrow$ extremal freqs: $\omega_p = \frac{\pi}{3}$, $\omega_s = \frac{\pi}{2}$, $\omega = \pi$

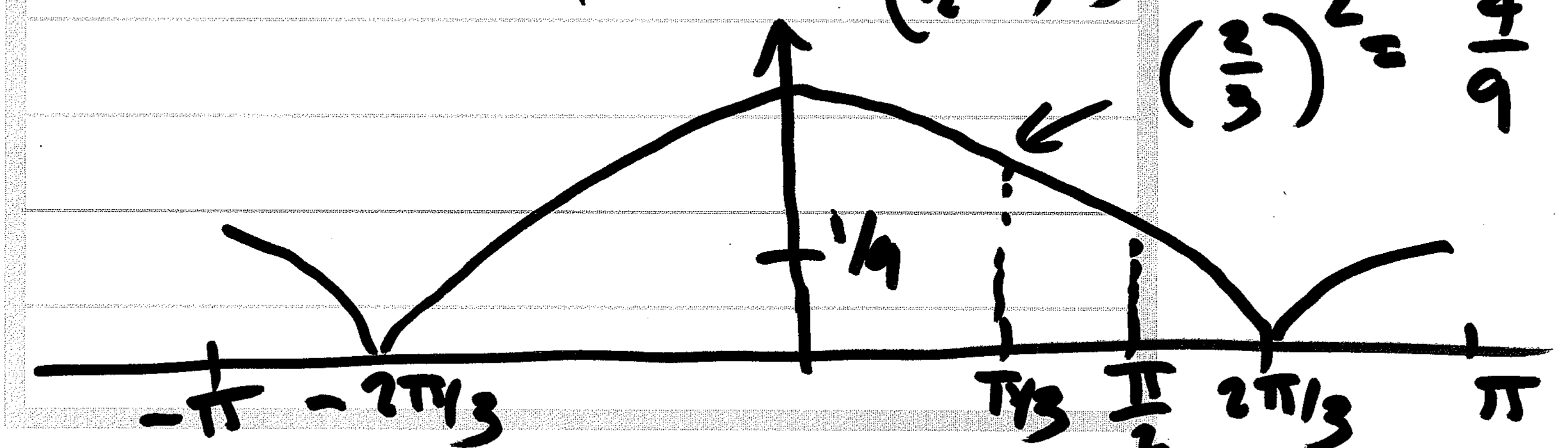
(e) $h_2[n] = \frac{1}{3}\{\underset{\uparrow}{1}, 1, 1\} * \frac{1}{3}\{\underset{\uparrow}{1}, 1, 1\}$ " $h_1[n] * h_1[n]$

$= \frac{1}{9}\{\underset{\uparrow}{1}, 2, 3, 2, 1\}$

$H_2(\omega) = H_1^2(\omega)$

$= \left\{\frac{1}{3}\frac{\sin(\frac{3}{2}\omega)}{\sin(\frac{1}{2}\omega)}\right\}^2 e^{-j2\omega}$

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$



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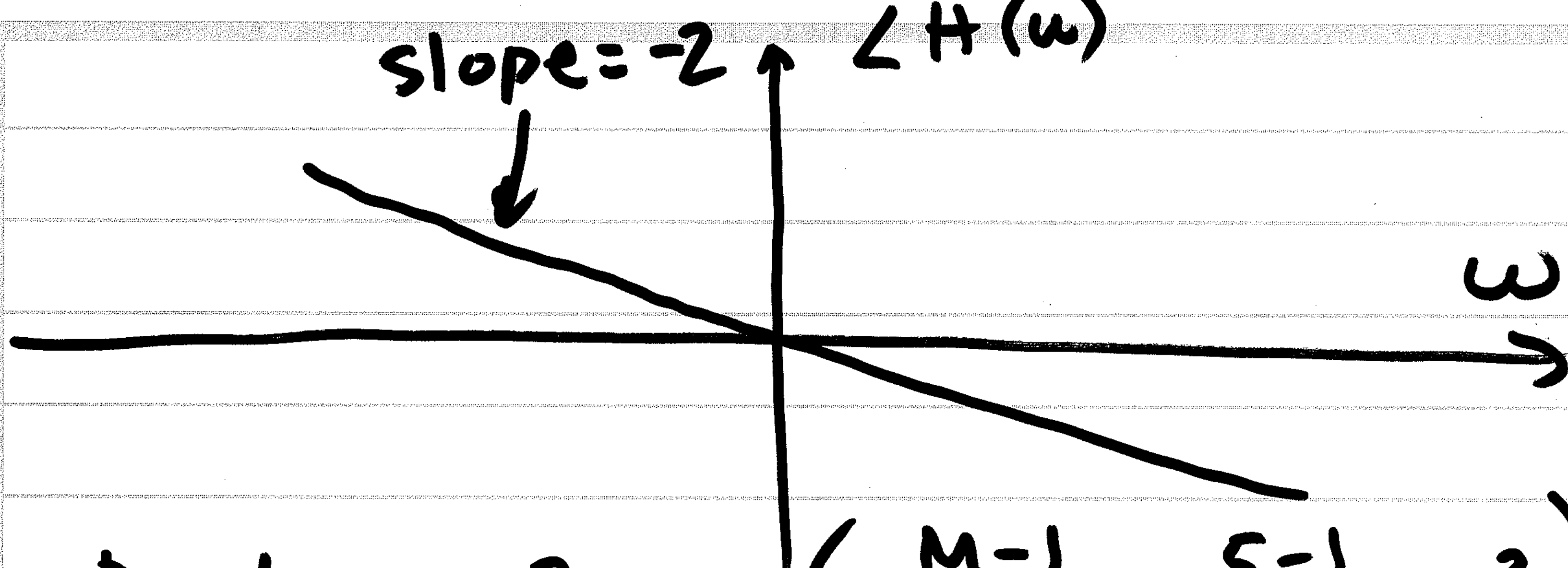

Delay = 2 $\left(\frac{M-1}{2} = \frac{5-1}{2} = 2\right)$

h) $\delta_1 = 1 - \frac{4}{9} = \frac{5}{9}$

(i) $\delta_2 = \frac{1}{9}$

Prob. 2 sol$^{\underline{n}}$:

$$s = \frac{z-1}{z+1} \Rightarrow z = \frac{1+s}{1-s} = \frac{1+s}{1-s}$$

(a) $$z_1 = \frac{1+s_1}{1-s_1} = \frac{1+e^{j2\pi/3}}{1-e^{j2\pi/3}}$$

$$= \frac{\cos\left(\frac{\pi}{3}\right)}{j\sin\left(\frac{\pi}{3}\right)} = -j\frac{1}{\tan\left(\frac{\pi}{3}\right)} = -j\frac{1}{\sqrt{3}}$$

$\Rightarrow z_3 = +j\frac{1}{\sqrt{3}}$ (similarly)

$$z_2 = \frac{1+(-1)}{1-(-1)} = 0$$

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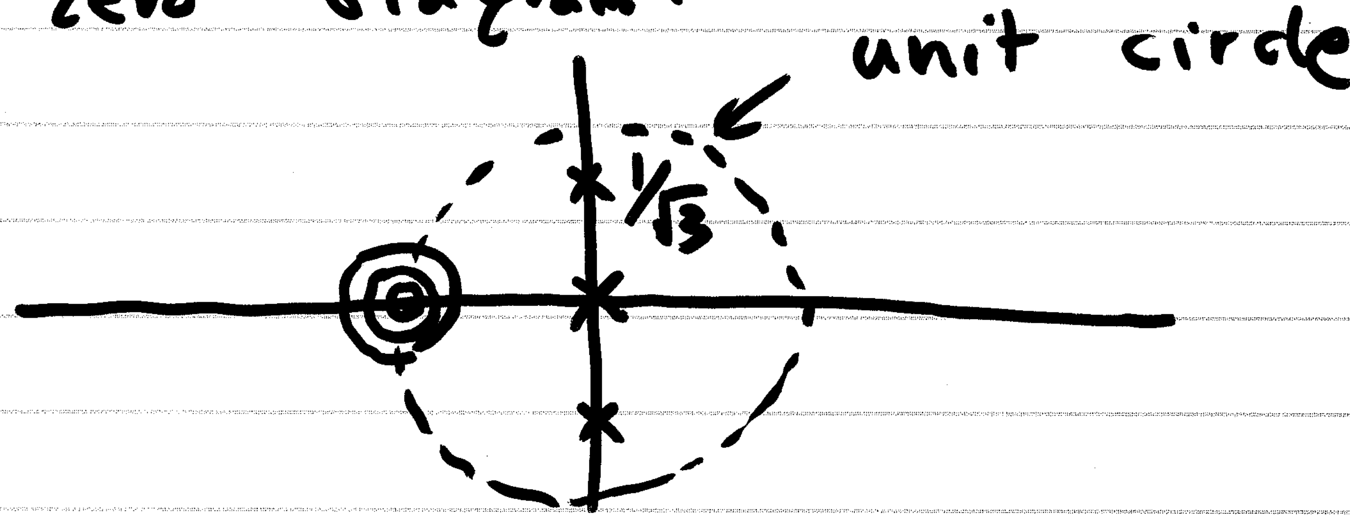
(b) pole-zero diagram:



(c) poles inside unit circle ⇒ stable!
stability is preserved thru bilinear transform ⇒ stable analog mapped to stable digital filter

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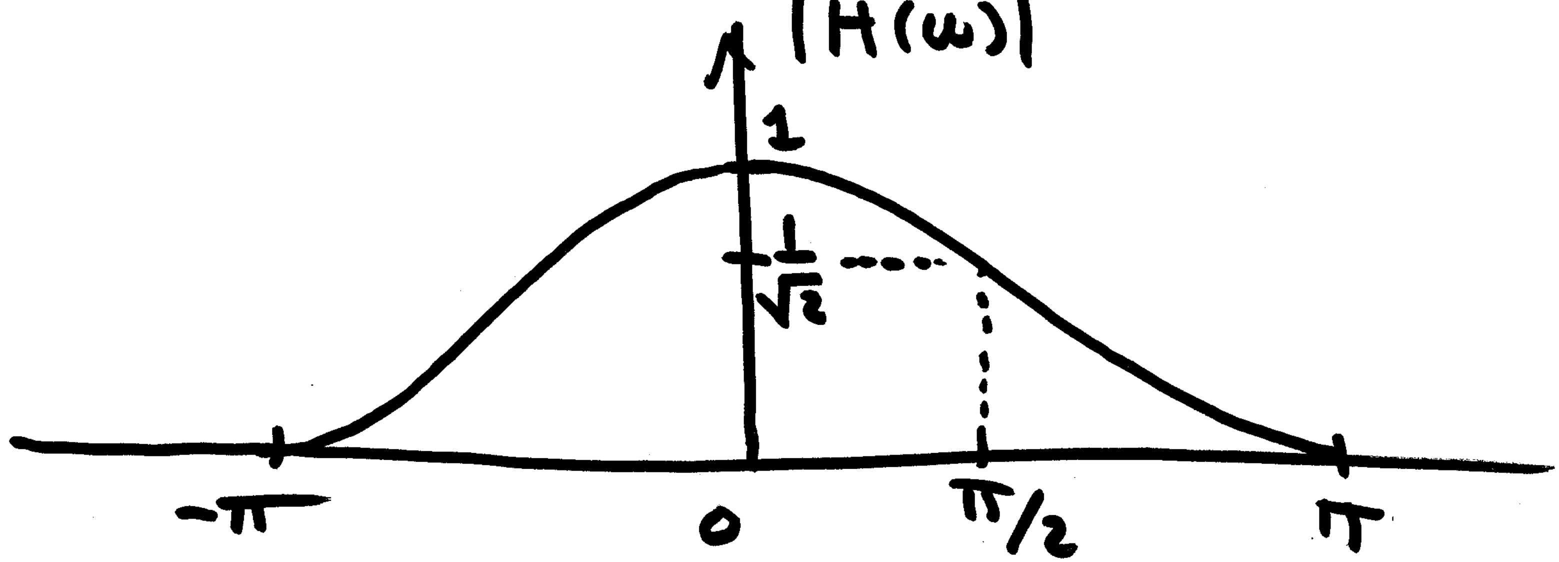
(d) Plot magnitude $\Rightarrow$ we know Butterworth filter (from textbook) monotonically decreases from 1 at $\Omega = 0$ to 0 as $\Omega \to \infty$ $\Rightarrow$ bilinear transform preserves monotonicity



(f) 3 dB point for Butterworth given $\Omega = 1$
Thus; from textbook (and derived in class $\omega = 2\tan^{-1}\left(\frac{\Omega}{c}\right)$ $\Rightarrow$ $c = 1$ here

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(f) cont. $\omega_c = 2\tan^{-1}(1) = 2\,\frac{\pi}{4} = \frac{\pi}{2}$

(e) $x[n] = e^{j0\cdot n} + e^{j\frac{\pi}{2}n} + e^{j\pi n}$

$$y[n] = H(0)\,e^{j0\cdot n} + H\left(\tfrac{\pi}{2}\right)e^{j\frac{\pi}{2}n} + H(\pi)\,e^{j\pi n}$$

$$= 1 + \frac{1}{\sqrt{2}}\,e^{j\left(\frac{\pi}{2}n+\theta\right)} + 0$$

$$= 1 + \frac{1}{\sqrt{2}}\,(j)^n\,e^{j\theta}$$

(g)

$$H(s) = \frac{1}{\left(s - e^{j\frac{2\pi}{3}}\right)(s+1)\left(s - e^{-j2\pi/3}\right)}$$

$$H(z) = H(s)\Big|_{s = \frac{z-1}{z+1}}$$

$$\frac{(z+1)^3}{(z+1)^3}\,\frac{1}{\left(\frac{z-1}{z+1} - e^{j\frac{2\pi}{3}}\right)\left(\frac{z-1}{z+1} + 1\right)\left(\frac{z-1}{z+1} - e^{-j2\pi/3}\right)}$$

$$= \frac{(z+1)^3}{\left(z-1 - e^{j2\pi/3}(z+1)\right)\left(z-1+z+1\right)\left(z-1 - e^{-j2\pi/3}(z+1)\right)}$$

(8)

$$H(z) = \frac{\frac{1}{6}\left(z^3 + 3z^2 + 3z^2 + 1\right)}{z\left(z^2 + \frac{1}{3}\right)}$$

$$y[n] = -\frac{1}{3}\,y[n-2] + \frac{1}{6}\,x[n]$$

$$+ \frac{1}{2}\,x[n-1] + \frac{1}{2}\,x[n-2] + \frac{1}{6}\,x[n-3]$$