ECE 538 Digital Signal Processing I Exam 3 Fall 2004
Session 33 Live: 10 Nov. 2004

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Symmetric FIR filter design.	50
2.	IIR Filter Design Via Bilinear Transform	50

Problem 1. [50 points]

A symmetric FIR lowpass filter of length $N = 3$ is designed via the Parks-McClellan algorithm. The passband edge is $\omega_p = \frac{\pi}{3}$ and the stopband edge is $\omega_p = \frac{\pi}{2}$.

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2] \tag{1}$$

(a) Plot the magnitude of the frequency response of this filter over $-\pi < \omega < \pi$. Show as much detail as possible. (Recall that the frequency response is the DTFT of the impulse response of the filter.) The following values of the sine function may be useful:

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}; \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}; \quad \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}; \quad \sin\left(\frac{\pi}{2}\right) = 1; \quad \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}; \quad \sin(\pi) = 0$$

(b) Plot the phase of the frequency response of this filter over $-\pi < \omega < \pi$.

  (i) Is the phase linear over the passband?

  (ii) What is the delay of this filter (in discrete time units)?

(c) The frequency response at $\omega = 0$ is one. What is the maximum deviation from one over the passband? That is, what is the passband ripple δ_1?

(d) What is the maximum absolute deviation from zero over the stopband? That is, what is the stopband ripple δ_2?

(e) How many extremal frequencies are there? What are the extremal frequencies?

Consider that we create a new filter by convolving the impulse response of the filter described by Equation (1) above with itself. This yields a new symmetric FIR filter described by the following difference equation:

$$y[n] = \frac{1}{9}x[n] + \frac{2}{9}x[n-1] + \frac{3}{9}x[n-2] + \frac{2}{9}x[n-3] + \frac{1}{9}x[n-4] \tag{2}$$

(f) Plot the magnitude of the frequency response of this new filter over $-\pi < \omega < \pi$. Show as much detail as possible.

(g) Plot the phase of the frequency response of the new filter over $-\pi < \omega < \pi$.

  (i) Is the phase linear over the passband?

  (ii) What is the delay of this new filter (in discrete time units)?

(h) The frequency response at $\omega = 0$ is one. For the new filter, what is the maximum deviation from one over the passband? That is, what is the passband ripple δ_1?

(i) For the new filter, what is the maximum absolute deviation from zero over the stopband? That is, what is the stopband ripple δ_2?

Problem 2. [50 points]

For the problem below, you might find the following values of the tangent function useful:

$$\tan\left\{\frac{\pi}{4}\right\} = 1; \qquad \tan\left\{\frac{\pi}{3}\right\} = \sqrt{3}; \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

If the values $1/\sqrt{3}$ or $1/\sqrt{2}$ show up in the problem, just carry them along.

A digital IIR filter is designed from an analog Butterworth filter via the Bilinear Transform method through the transformation $s = \frac{z-1}{z+1}$. A canonical analog Butterworth filter with a 3 dB cut-off frequency of $\Omega = 1$ rads/sec has three poles at the following locations in the s-plane:

$$s_1 = e^{j\frac{2\pi}{3}}; \quad s_2 = -1; \quad s_3 = e^{-j\frac{2\pi}{3}}$$

The analog Butterworth filter has a gain of one (unity) at DC.

(a) Determine the locations of the three poles of the resulting digital IIR filter.

(b) Plot a pole-zero diagram for the resulting digital filter. Be sure to show where the zeros are located, as well as the poles.

(c) Is the resulting digital filter stable or unstable? Explain your answer.

(d) Plot the magnitude of the frequency response of the digital IIR filter over $-\pi < \omega < \pi$. Show as much detail as possible. You must clearly indicate what the magnitude of the frequency response is at the following three digital frequencies: $\omega = 0$, $\omega = \frac{\pi}{2}$, and $\omega = \pi$.

(e) Find the output of the digital filter to the following input which is a sum of sinewaves "turned on" for all n.

$$x[n] = 1 + (j)^n + (-1)^n$$

(f) For the digital filter, what frequency is the 3 dB point? That is, at what frequency does the magnitude drop from one at $\omega = 0$ to a value equal to $\frac{1}{\sqrt{2}}$?

(g) Write the difference equation for the resulting digital filter.