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Prob. 1

(a)

$$S_{xx}(\omega) = \frac{2}{\left| 1 - \frac{1}{2} e^{-j\omega} + \frac{1}{4} e^{-j2\omega} - \frac{1}{8} e^{-j3\omega} \right|^2}$$

(b) Since AR(3) process:

$a_3(1) = a_1 = -\frac{1}{2}$ $\quad$ $a_3(2) = a_2 = \frac{1}{4}$

$a_3(3) = a_3 = -\frac{1}{8}$ $\quad$ $\mathcal{E}_{min}^{(3)} = \sigma_w^2 = 2$

(c) Since AR(3) process:

$a_4(1) = -\frac{1}{2}$ $\quad$ $a_4(2) = \frac{1}{4}$

$a_4(3) = -\frac{1}{8}$ $\quad$ $a_4(4) = 0$

$$\mathcal{E}_{min}^{(4)} = \mathcal{E}_{min}^{(3)} = \sigma_w^2 = 2$$

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Sol'n to Prob. 2

(a) $$x_8[n] = \sum_{l=0}^{\infty} \left(.9e^{j\frac{\pi}{4}}\right)^{(n+l8)}$$

$n = 0, 1, \ldots, 7$

$$= \left(.9e^{j\frac{\pi}{4}}\right)^n \sum_{l=0}^{\infty} \left(.9^8 e^{j\frac{\pi 8}{4}}\right)^l$$

$$= \left(.9e^{j\frac{\pi}{4}}\right)^n \frac{1}{1 - .9^8 e^{j2\pi}}$$

$$= 1.7558 \, (.9)^n e^{j\frac{\pi}{4}n}$$

$n = 0, 1, \ldots, 7$

(b) $$\frac{x_8[n]}{x_8[0]} = .9^n e^{j\frac{\pi}{4}n} = x[n]$$

- time-domain aliasing only manifests itself as a multiplicative scalar

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Prob. 2 (cont.)

(c) Use superposition

$$y[n] = \left\{ (.9 e^{j\frac{\pi}{4}})^n + (.9 e^{-j\frac{\pi}{4}})^n \right\} u[n]$$

It follows from (a) and superposition

$$y_8[n] = 1.7558 \; (.9)^n e^{j\frac{\pi}{4}n}$$

$$+ 1.7558 \; (.9)^n e^{-j\frac{\pi}{4}n}$$

$$= (1.7558) \; 2 \; (.9)^n \cos\left(\frac{\pi}{4}n\right)$$

$n = 0, 1, \ldots, 7$

(d) same in this case because of specific frequency $\omega = \pi/4$

If the frequency was $\pi/3$, for example, then $y_8[n] \neq y[n]$

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Prob. 3

(a)

$$v[n] \longrightarrow \overset{\text{FIR}}{\boxed{1-2z^{-1}}} \longrightarrow \overset{\text{IIR}}{\boxed{\frac{1}{1-\frac{1}{2}z^{-1}}}} \longrightarrow x[n]$$

$\sigma_n^2 = 1$

$\Downarrow$

$$r_{xx}[m] = \sigma_n^2 \; r_{FIR}[m] * r_{IIR}[m]$$

$$= \left[\{\underset{\uparrow}{1}, -2\} * \{-2, \underset{\uparrow}{1}\} \right] * \frac{1}{1-\left(\frac{1}{2}\right)^2} \left(\frac{1}{2}\right)^{|m|}$$

$$= \{-2, \underset{\uparrow}{5}, -2\} * \frac{4}{3} \left(\frac{1}{2}\right)^{|m|}$$

$$= \frac{4}{3} \left\{ -2\left(\frac{1}{2}\right)^{|m+1|} + 5\left(\frac{1}{2}\right)^{|m|} - 2\left(\frac{1}{2}\right)^{|m-1|} \right\}$$

for $m \geq 1$:

$$r_{xx}[m] = \frac{4}{3} \left\{ -2\left(\frac{1}{2}\right)^{m+1} + 5\left(\frac{1}{2}\right)^{m} - 2\left(\frac{1}{2}\right)^{m-1} \right\}$$

$$= \frac{4}{3} \left(\frac{1}{2}\right)^{m} \left\{ -1 + 5 - 4 \right\} = 0 \;\; !!$$

$r_{xx}[0] = 4\,\delta[m]$; $r_{xx}[m] = 0$ for $|m| \geq 1$

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(b) $S_{xx}(\omega) = \text{DTFT}\{ r_{xx}[m] \}$

$$= \text{DTFT}\{ 4\,\delta[m] \}$$

$$= 4 \quad \text{for all } \omega$$

Alternatively:

$$S_{xx}(\omega) = \sigma_w^2 \, |H(\omega)|^2$$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = \frac{e^{j\omega} - 2}{e^{j\omega} - \frac{1}{2}}$$

$$|H(\omega)|^2 = \frac{e^{j\omega} - 2}{e^{j\omega} - \frac{1}{2}} \cdot \frac{e^{-j\omega} - 2}{e^{-j\omega} - \frac{1}{2}}$$

$$= \frac{1 + 4 - 4\cos(\omega)}{1 + \frac{1}{4} - \cos(\omega)} \cdot \frac{4}{4}$$

$$= \frac{4\,(5 - 4\cos(\omega))}{5 - 4\cos(\omega)}$$

$$= 4 \quad \text{for all } \omega$$ Check!

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Prob. 3 (cont.)

$$r_{xx}[m] = 4\,\delta[m]$$

(c)

$$a_1(1) = \frac{-r_{xx}[1]}{r_{xx}[0]} = \frac{0}{4} = 0$$

$\Rightarrow$ all-pass filter

$\Rightarrow$ white noise in

$\Rightarrow$ white noise out

$\Rightarrow$ can't predict white noise

$$\mathcal{E}_{min}^{(1)} = 4 = r_{xx}[0]$$

(d)

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} a_2(1) \\ a_2(2) \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a_2(1) = 0$ $a_2(2) = 0$ $\Rightarrow$ see above

$$\mathcal{E}_{min}^{(2)} = r_{xx}[0] = 4$$