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P1)

$$x_i(m) = \frac{1}{16} \cdot \sum_{k=0}^{15} X_i(k)\, e^{j\frac{2\pi k}{16} m} \qquad m = 0, 1, \dots, 15 \qquad (i = 1, 2)$$

$$X_1(k) = \frac{1}{2}\left\{ X(k) + X^*(16-k) \right\}$$

$$= 4 \cdot \left\{ \delta(k-2) + \cancel{\delta(k-5)} - \cancel{\delta(k-11)} + \delta(k-14) + \delta(14-k) + \cancel{\delta(11-k)} - \cancel{\delta(5-k)} + \delta(2-k) \right\}$$

$$= 8 \cdot \left\{ \delta(k-2) + \delta(k-14) \right\}$$

$$\therefore \quad x_1(m) = \frac{1}{2} \cdot \left\{ e^{j\frac{\pi}{4} m} + \underbrace{e^{j\frac{7\pi}{4} m}}_{e^{-j\frac{\pi}{4} n}} \right\} = \cos\left(\frac{\pi}{4} n\right) \qquad m = 0, 1, \dots, 15$$

$$X_2(k) = \frac{1}{2j}\left\{ X(k) - X^*(16-k) \right\}$$

$$= 8 \cdot \left\{ \delta(k-5) - \delta(k-11) \right\}$$

$$\therefore \quad x_2(m) = \frac{1}{2j}\left\{ e^{j\frac{5\pi}{8} m} - \underbrace{e^{j\frac{11\pi}{8} m}}_{e^{j\left(\frac{11\pi}{8} n - \frac{16\pi}{8} n\right)}} \right\} \qquad m = 0, 1, \dots, 15$$

$$= \sin\left(\frac{5\pi}{8} n\right)$$

P2) a) $$x_8(n) = \sum_{l=-\infty}^{\infty} (0.9)^{n-8l}\, u(n-8l)$$

$$= (0.9)^n \cdot \sum_{l=-\infty}^{\infty} (0.9)^{-8l}\, u(n-8l) \underset{n=0,1,\dots,7}{\uparrow{=}} (0.9)^n \cdot \sum_{l=-\infty}^{0} (0.9)^{-8l}$$

$$= (0.9)^n \cdot \sum_{l=0}^{\infty} \left(0.9^8\right)^l$$

$$= (0.9)^n \cdot \frac{1}{1-(0.9)^8} = \underbrace{1.7558\,(0.9)^n}_{(n=0,1,\dots,7)}$$ (Periodic with period 8)

b) $$\tilde{x}_8(n) = \frac{x_8(n)}{x_8(0)} = x(n) \qquad n = 0,1,\dots,7$$

aliasing (in this case) corresponds to a multiplicative factor. Hence, by the above normalization ($x(0) = \tilde{x}_8(0) = 1$) we make $x(n) = \tilde{x}_8(n)$ over the period $n = 0,\dots,7$ (of $\tilde{x}_8$).

c) By part a):

$$y_8(n) = (0.9)^n \cdot \frac{1}{1-(0.9)^8} + (0.8)^n \cdot \frac{1}{1-(0.8)^8}$$

$\therefore$

$$y_8(n) = 1.7558\,(0.9)^n + 1.2016\,(0.8)^n \qquad n = 0,1,\dots,7$$ (Periodic with period 8)

d) $$\tilde{y}_8(n) = y_8(n)/y_8(0) = \frac{1}{1.7558+1.2016}\left(1.7558\,(0.9)^n + 1.2016\,(0.8)^n\right)$$

$\neq y(n)$ $(n = 0,\dots,7)$ since aliasing is now not a multiplicative factor

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P3) a) $$\begin{pmatrix} r_{xx}(0) & r_{xx}(-1) \\ r_{xx}(1) & r_{xx}(0) \end{pmatrix} \begin{pmatrix} a_1^{(2)} \\ a_2^{(2)} \end{pmatrix} = -\begin{pmatrix} r_{xx}(1) \\ r_{xx}(2) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} a_1^{(2)} \\ a_2^{(2)} \end{pmatrix} = -\begin{pmatrix} 3 \\ 3/2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} a_1^{(2)} \\ a_2^{(2)} \end{pmatrix} = -\frac{1}{16-9} \cdot \begin{pmatrix} 4 & -3 \\ -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3/2 \end{pmatrix} = -\frac{1}{7} \cdot \begin{pmatrix} 15/2 \\ -3 \end{pmatrix}$$

$$\varepsilon_{min}^{(2)} = r_{xx}(0) + a_1^{(2)} r_{xx}(-1) + a_2^{(2)} r_{xx}(-2)$$

$$= 4 - \frac{15}{14} \cdot 3 + \frac{3}{7} \cdot \frac{3}{2} = 10/7$$

b) $$x(n) = -\sum_{k=1}^{p} \tilde{a}_k \, x(n-k) + \sum_{k=0}^{q} b_k \, w(n-k)$$

$\therefore \; q = 1$

$$\therefore \quad r_{xx}(2) = -\tilde{a}_1 \, r_{xx}(1) \Rightarrow \tilde{a}_1 = -\frac{3/2}{3} = -1/2$$

$$\therefore \quad a_1 = -\tilde{a}_1 = 1/2$$

c)

$$r_{xx}(0) = a_1 r_{xx}(-1) + \sigma_w^2 \left(h(0) b_0 + h(1) b_1 \right)$$

$$r_{xx}(1) = a_1 r_{xx}(0) + \sigma_w^2 h(0) b_1$$

ie:
$$4 = \frac{1}{2} \cdot 3 + h(0)\, b_0 + h(1)\, b_1$$
$$3 = \frac{1}{2} \cdot 4 + h(0)\, b_1$$

also:
$$H(z) = \frac{X(z)}{W(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0}{1 - a_1 z^{-1}} + \frac{b_1 z^{-1}}{1 - a_1 z^{-1}}$$

$$\therefore \quad h(n) = b_0\, (a_1)^n\, u(n) + b_1\, (a_1)^{n-1}\, u(n-1)$$
(causal)

ie $\quad h(0) = b_0 \quad ; \quad h(1) = b_0 a_1 + b_1$

Hence:
$$\left.\begin{array}{l} b_0 b_1 = 1 \\ b_0^2 + b_1^2 = 2 \end{array}\right\} \Rightarrow (b_0 - b_1)^2 = 2 - 2 \cdot 1 = 0$$
$$\therefore \quad b_0 = b_1$$
$$\therefore \quad b_0 = b_1 = 1$$

d)
$$S_{xx}(w) = \sum_{m=-\infty}^{\infty} r_{xx}(m)\, e^{-jmw}$$

$$= -2 + 6 \cdot \left\{ -1 + \sum_{m=0}^{\infty} \left(\frac{e^{-jw}}{2} \right)^m + \sum_{m=0}^{\infty} \left(\frac{e^{jw}}{2} \right)^m \right\}$$

$$= -2 + 6 \cdot \left\{ -1 + \frac{1}{1 - \frac{e^{-jw}}{2}} + \frac{1}{1 - \frac{e^{jw}}{2}} \right\}$$

or:
$$= \frac{\left| 1 + e^{-jw} \right|^2}{\left| 1 - \frac{e^{-jw}}{2} \right|^2} = s_w^2 \cdot \frac{\left| \sum_{k=0}^{q} b_k e^{-jkw} \right|^2}{\left| 1 + \sum_{k=1}^{p} \tilde{a}_k e^{-jkw} \right|^2}$$