EE538 Exam 3 Fall 2002
Digital Signal Processing I Session 41, 2002

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators **are** allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

No.	Topic(s) of Problem	Points
1.	DFT of real-valued sequences, DFT properties, and standard DFT pairs	30
2.	Sampling of DTFT and Time-Domain Aliasing	30
3.	Autoregressive spectral estimation; basics of AR, MA, and ARMA processes.	40

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Problem 1. [30 points]

Let $x_1[n]$ and $x_2[n]$ denote two (different) real-valued sequences of length $N = 16$ that are only nonzero for $0 \leq n \leq 15$. Consider that we create a complex-valued sequence as

$$x[n] = x_1[n] + jx_2[n]$$

The 16-pt DFT of $x[n]$, denoted $X_{16}(k)$, is given by

$$X_{16}(k) = 8\delta(k-2) + 8\delta(k-5) - 8\delta(k-11) + 8\delta(k-14)$$

Using properties of the DFT and standard DFT pairs, determine *closed-formed* expressions for both $x_1[n]$ and $x_2[n]$. A *closed-form* expression contains NO summations and it is NOT a listing of numbers.

Problem 2. [30 points]

(a) Let $X_8(k) = X(2\pi k/8)$, where $X(\omega)$ is the DTFT of the sequence

$$x[n] = (0.9)^n u[n] \overset{DTFT}{\longleftrightarrow} X(\omega) = \frac{1}{1 - 0.9e^{-j\omega}}$$

That is, $X_8(k)$ is what we obtain by sampling $X(\omega)$ at $N = 8$ equi-spaced points in the interval $0 \leq \omega < 2\pi$. Theory derived in class and in the textbook dictates that the 8-pt inverse DFT of $X_8(k)$ may be expressed as

$$x_8[n] = \sum_{\ell=-\infty}^{\infty} x[n - \ell 8]\{u[n] - u[n-8]\} \underset{8}{\overset{DFT}{\longleftrightarrow}} X_8(k) = \frac{1}{1 - 0.9e^{-j2\pi k/8}}; k = 0, 1, ..., 7$$

Determine a simple, closed-form expression for $x_8[n]$. A *closed-form* expression contains NO summations and it is NOT a listing of numbers. *Hint:*

$$\frac{1}{1 - (.9)^8} = 1.7558$$

(b) Consider normalizing $x_8[n]$ so that it's first value is one, $\tilde{x}_8[n] = x_8[n]/x_8[0]$, $n = 0, 1, ..., 7$. Compare $\tilde{x}_8[n]$ and $x[n]$ over $n = 0, 1, ..., 7$. Are they the same or different? Briefly explain your answer as to why or why not they are the same.

(c) Let $Y_8(k) = Y(2\pi k/8)$, where $Y(\omega)$ is the DTFT of the sequence

$$y[n] = (0.9)^n u[n] + (0.8)^n u[n]$$

That is, $Y_8(k)$ is what we obtain by sampling $Y(\omega)$ at $N = 8$ equi-spaced points in the interval $0 \le \omega < 2\pi$. Theory derived in class and in the textbook dictates that the 8-pt inverse DFT of $Y_8(k)$ may be expressed as

$$y_8[n] = \sum_{\ell=-\infty}^{\infty} y[n-\ell 8]\{u[n]-u[n-8]\} \overset{DFT}{\underset{8}{\longleftrightarrow}} Y_8(k) = \frac{1}{1-0.9e^{-j2\pi k/8}} + \frac{1}{1-0.8e^{-j2\pi k/8}}$$

Determine a simple, closed-form expression for $y_8[n]$. A *closed-form* expression contains NO summations and it is NOT a listing of numbers. *Hint:*

$$\frac{1}{1-(.8)^8} = 1.2016$$

(d) Consider normalizing $y_8[n]$ so that it's first value is one, $\tilde{y}_8[n] = y_8[n]/y_8[0]$, $n = 0, 1, ..., 7$. Compare $\tilde{y}_8[n]$ and $y[n]$ over $n = 0, 1, ..., 7$. Are they the same or different? Briefly explain your answer as to why or why not they are the same.

Problem 3. [40 points]

Consider the ARMA(1,1) process generated via the difference equation

$$x[n] = a_1 x[n-1] + b_0 w[n] + b_1 w[n-1]$$

where $w[n]$ is a stationary white noise process with variance $\sigma_w^2 = 1$. The autocorrelation sequence $r_{xx}[m] = E\{x[n]x[n-m]\}$ is given by the following closed-form expression which holds for m from $-\infty$ to ∞:

$$r_{xx}[m] = 6\left(\frac{1}{2}\right)^{|m|} - 2\delta[m]$$

(a) Consider that the power spectrum of the ARMA(1,1) process $x[n]$ is estimated via AR spectral estimation according to

$$S_{xx}(\omega) = \frac{\mathcal{E}_{min}^{(2)}}{|1 + a_1^{(2)} e^{-j\omega} + a_2^{(2)} e^{-j2\omega}|^2}$$

Determine the respective numerical values of the optimum second-order linear prediction coefficients $a_1^{(2)}$ and $a_2^{(2)}$ and the value of the corresponding minimum mean-square error $\mathcal{E}_{min}^{(2)}$.

(b) Determine the numerical value of the coefficient a_1 in the difference equation above defining the LTI system that the white noise was passed through to generate the ARMA(1,1) process $x[n]$.

(c) Determine the respective numerical values of the coefficients b_0 and b_1 in the difference equation above defining the LTI system that the white noise was passed through to generate the ARMA(1,1) process.

(d) Determine a simple closed-form expression for the spectral density for $x[n]$, $S_{xx}(\omega)$, which may be expressed as the DTFT of $r_{xx}[m]$:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}[m] e^{-jm\omega}$$