EE538 **Exam 3** **Fall 2001**
Digital Signal Processing I **29 November 2001**

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators **not** allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

No.	**Topic(s) of Problem**	**Points**
1.	Autoregressive (AR) Spectral Estimation	40
2.	Sum of Sinewaves Spectral Analysis	30
3.	Spectral Characteristics of ARMA Processes	30

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Problem 1. [30 points]

Consider the autoregressive AR(2) process generated via the difference equation

$$x[n] = \frac{7}{12}x[n-1] - \frac{2}{12}x[n-2] + w[n]$$

where $w[n]$ is a stationary white noise process with variance $\sigma_w^2 = 35/6$. (The value of σ_w^2 was chosen so that the autocorrelation values requested in part (a) below are whole numbers.)

(a) Determine the numerical values of $r_{xx}[0]$, $r_{xx}[1]$, $r_{xx}[2]$, where $r_{xx}[m]$ is the autocorrelation sequence $r_{xx}[m] = E\{x[n]x[n-m]\}$.

(b) Determine a simple closed-form expression for the spectral density for $x[n]$, $S_{xx}(\omega)$, which may be expressed as the DTFT of $r_{xx}[m]$:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}[m]e^{-jm\omega}$$

(c) Consider the first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the corresponding minimum mean-square error.

(d) Consider the third-order predictor

$$\hat{x}[n] = -a_3(1)x[n-1] - a_3(2)x[n-2] - a_3(3)x[n-3]$$

Determine the numerical values of the optimum predictor coefficients $a_3(1)$, $a_3(2)$, and $a_3(3)$ and the corresponding minimum mean-square error.

Problem 2. [30 points]

Consider the discrete-time complex-valued random process defined for all n:

$$x[n] = e^{\jmath(\omega_1 n + \Theta_1)} + \sqrt{2}\, e^{\jmath(\omega_2 n + \Theta_2)}$$

where the respective frequencies, ω_1 and ω_2, of the two complex sinewaves are deterministic but unknown constants. Θ_1 and Θ_2 are independent random variables with each uniformly distributed over a 2π interval. The values of the autocorrelation sequence for $x[n]$, $r_{xx}[m] = E\{x[n]x^*[n-m]\}$, for three different lag values are given below.

$$r_{xx}[0] = 3, \quad r_{xx}[1] = -2 + j, \quad r_{xx}[2] = 1$$

(a) Determine the numerical values of ω_1 and ω_2. **You have to use what you've learned during the parametric spectral analysis portion of this course. You will be given no credit if you simply set up a system of equations to solve based on the form of $r_{xx}[m] = \sum_{i=1}^{p} A_i^2 e^{j\omega_i m}$ and solve this nonlinear system of equations.** Part (b) is on top of next page.

(b) Consider a second-order predictor

$$\hat{x}[n] = -a_2(1)x[n-1] - a_2(2)x[n-2]$$

Determine the numerical values of the optimum predictor coefficients $a_2(1)$ and $a_2(2)$, and the numerical value of the corresponding minimum mean-square error.

Problem 3. [30 points]

Consider the ARMA(1,1) process generated via the difference equation

$$x[n] = \frac{1}{2}x[n-1] + w[n] + w[n-1]$$

where $w[n]$ is a stationary white noise process with variance $\sigma_w^2 = 1$.

(a) Determine the numerical values of $r_{xx}[0]$, $r_{xx}[1]$, $r_{xx}[2]$, where $r_{xx}[m]$ is the autocorrelation sequence $r_{xx}[m] = E\{x[n]x[n-m]\}$. (Note that $r_{xx}[m]$ is the inverse DTFT of the spectral density $S_{xx}(\omega)$ asked for in Part (b) below, but there at least three different ways you can solve this part of the problem.)

(b) Determine a simple closed-form expression for the spectral density for $x[n]$, $S_{xx}(\omega)$, which may be expressed as the DTFT of $r_{xx}[m]$:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}[m]e^{-jm\omega}$$

(c) Consider the first-order predictor

$$\hat{x}[n] = -a_1(1)x[n-1]$$

Determine the numerical value of the optimum predictor coefficient $a_1(1)$ and the corresponding minimum mean-square error.