Solution to Exam 2 Fall 2006

Friday, October 22, 2010
1:23 PM

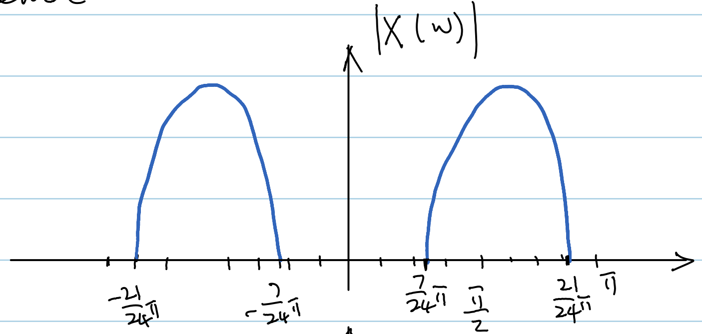
Problem 1:

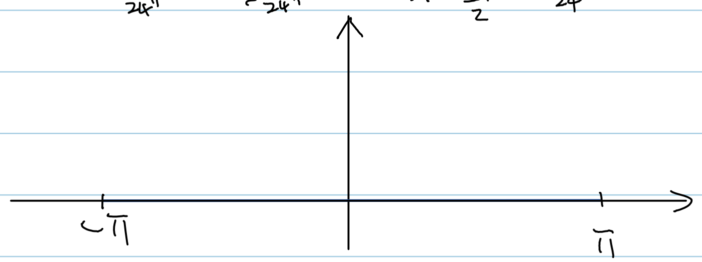
(a) Sampling at rate F_s will scale the analog frequency by F_s from $-\pi$ to π the edges of the band of $x_a(t)$ will be scaled to:

$$W \rightarrow W/F_s \cdot 2\pi = \frac{7}{8}\pi = \frac{21}{24}\pi$$

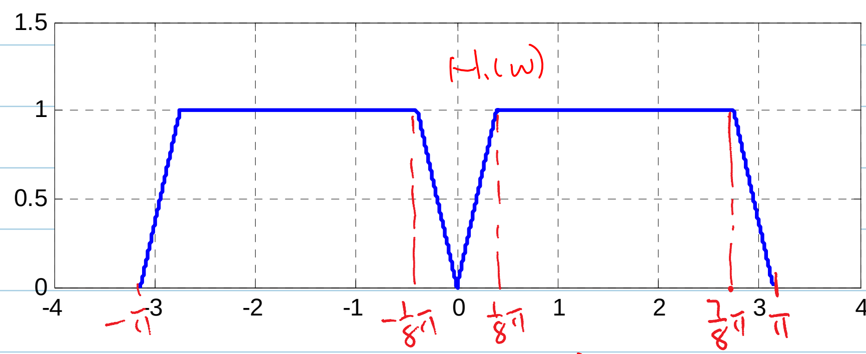
$$\frac{W}{3} \rightarrow W/F_s \cdot 2\pi = \frac{7}{24}\pi$$

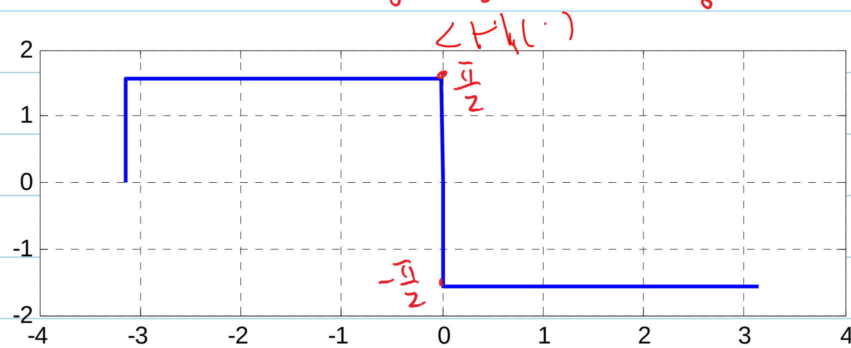
hence

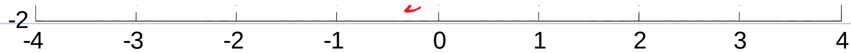




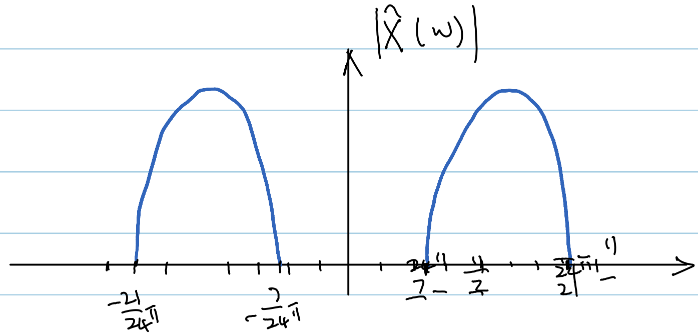
(b)

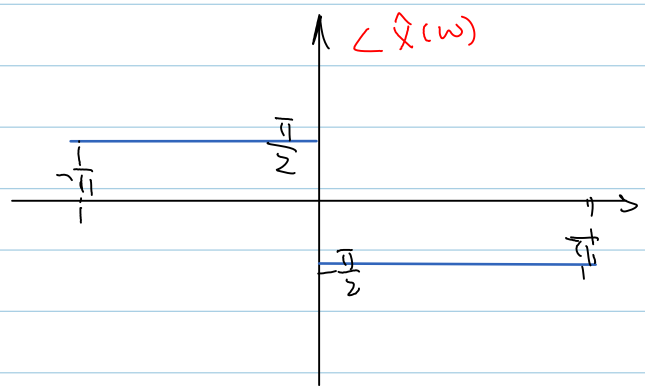




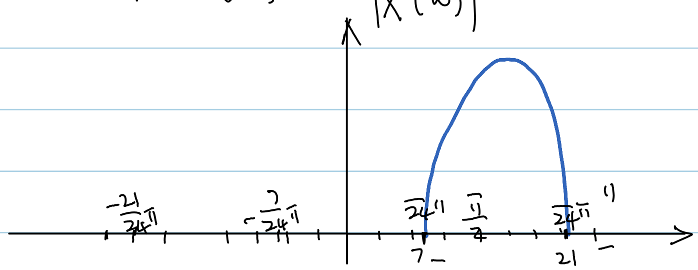


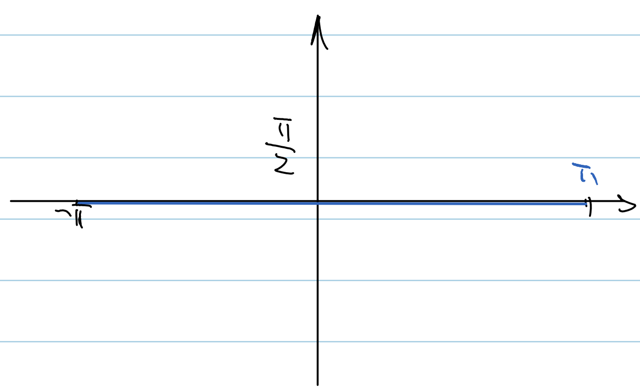
c) Since $X(\omega)$ is within the flat region of $H_i(\omega)$ hence $\hat{X}(\omega)$ is:



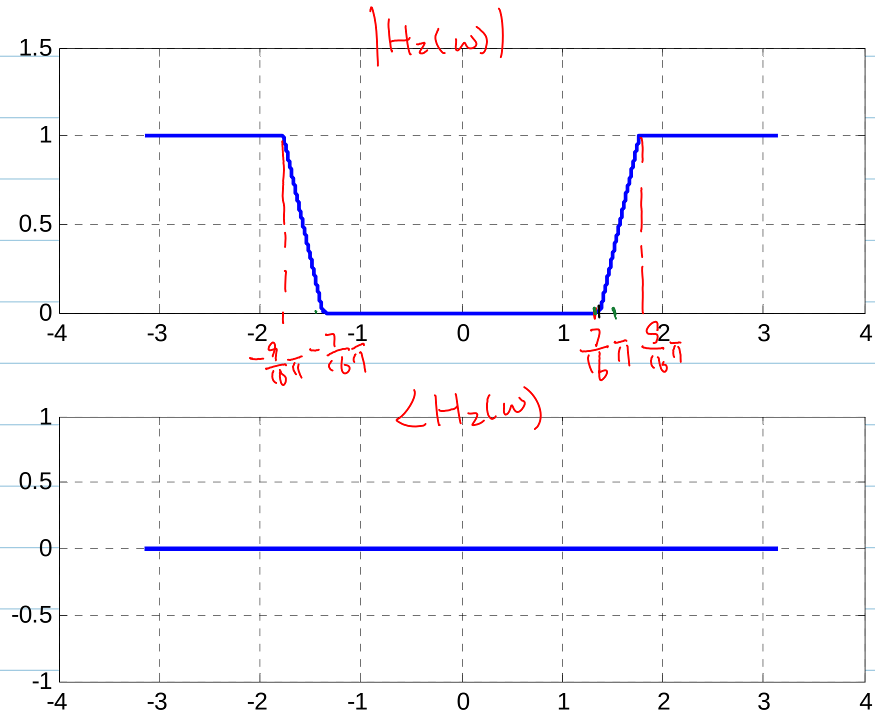


hence the Hilbert transform zeros out the negtive frequency part and doubles the positive one. and results in $|\hat{X}(\omega)|$

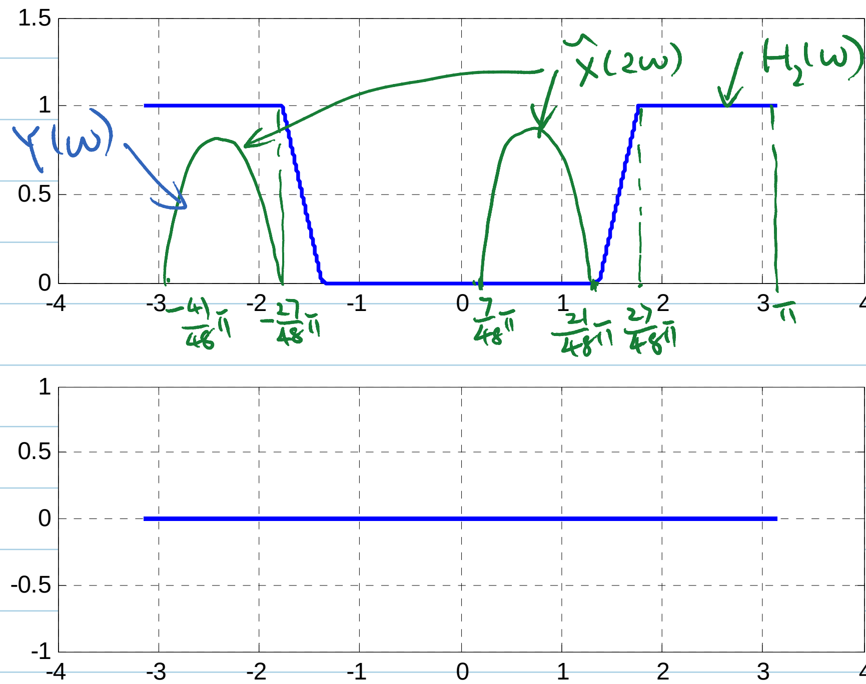




(d)



(e) zero inserting $\hat{x}(n)$ gives $\hat{X}(2\omega)$



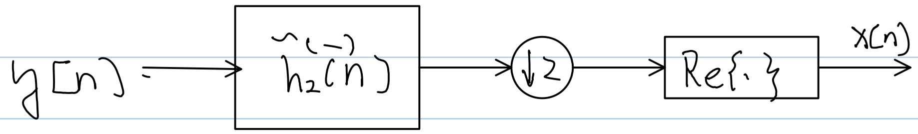
It is obvious in the plot, only

the replica in the band $[-\frac{47}{48}\pi, -\frac{23}{48}\pi)$ is left in $Y(w)$, which concludes the subbanding process

(f) $y[n] = \tilde{x}_a\left(\frac{n}{2F_s}\right) e^{j\pi n}$

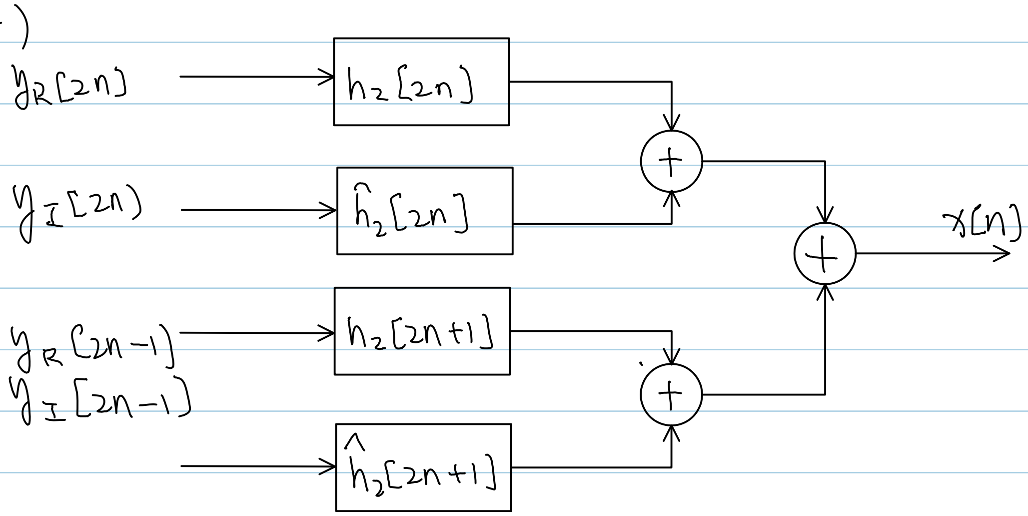
$$y[2n] = \tilde{x}_a\left(\frac{2n}{2F_s}\right) e^{j2\pi n}$$

$$= \tilde{x}[n] = x[n] + j[n]$$



where $\tilde{h}_2^{(-)}[n] = h_2[n] - j\,\hat{h}_2[n]$

(g)



Problem 2

sampling at $F_s = \frac{8}{3}W$ results in

the signal with band width

$$2\frac{W}{F_s}2\pi = 2\cdot\frac{3}{8}2\pi = 2\cdot\frac{3}{4}\pi$$

hence the discrete signal is in the band $\omega \in \bigcup_{k=-\infty}^{+\infty}\left(-\frac{3}{4}\pi+2k\pi, \frac{3}{4}\pi+2k\pi\right)$

(a) zero inserting by factor 4 compresses the signal in frequency and the resultant signal is in the band

$$\omega \in \bigcup_{k=-\infty}^{+\infty}\left(-\frac{3}{16}\pi+\frac{\pi}{2}k, \frac{3}{16}\pi+\frac{\pi}{2}k\right)$$

hence $\omega_{p1} = \frac{3}{16}\pi \quad \omega_{s1} = \frac{5}{16}\pi$

then the new signal is non-zero in

$$\omega \in \bigcup_{k=-\infty}^{+\infty}\left(-\frac{3}{16}\pi+2k\pi, \frac{3}{16}\pi+2k\pi\right)$$

(b) similarly further zero inserting results in signal in the interval.

$$\omega \in \bigcup_{k=-\infty}^{+\infty}\left(-\frac{3}{64}\pi+\frac{\pi}{2}k, \frac{3}{64}\pi+\frac{\pi}{2}k\right)$$

hence

$$\omega_{p2} = \frac{3}{64}\pi \qquad \omega_{s2} = \left(-\frac{3\pi}{64}+\frac{\pi}{2}\right) = \frac{29}{64}\pi$$