NAME: **25 Oct. 2019**

ECE 538 Digital Signal Processing I Exam 2 Fall 2019

Cover Sheet

WRITE YOUR NAME ON THIS COVER SHEET

Test Duration: 60 minutes.

Open Book but Closed Notes.

One (both sides) handwritten 8.5 in x 11 in crib sheet allowed

Calculators NOT allowed.

All work should be done in the space provided.

Clearly mark your answer to each part.

Continuous-Time Fourier Transform (Hz): $X(F) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$

Continuous-Time Fourier Transform Pair (Hz): $\mathcal{F}\left\{\frac{\sin(2\pi Wt)}{\pi t}\right\} = rect\left\{\frac{F}{2W}\right\}$ where $rect(x) = 1$ for $|x| < 0.5$ and $rect(x) = 0$ for $|x| > 0.5$.

Continuous-Time Fourier Transform Property: $\mathcal{F}\{x_1(t)x_2(t)\} = X_1(F) * X_2(F)$, where * denotes convolution, and $\mathcal{F}\{x_i(t)\} = X_i(F)$, $i = 1, 2$.

Relationship between DTFT and CTFT frequency variables in Hz: $\omega = 2\pi\frac{F}{F_s}$, where $F_s = \frac{1}{T_s}$ is the sampling rate in Hz

Problem 1. Consider the upsampler system below in Figure 1.

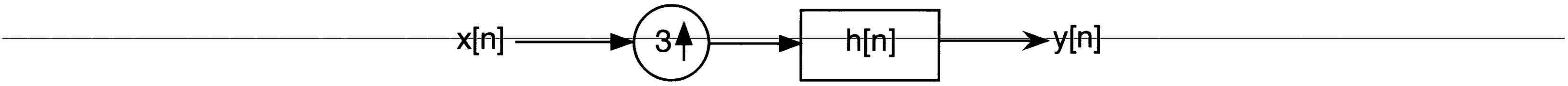


Figure 1.

(a) Draw block diagram of an efficient implementation of the upsampler system in Fig. 1.

(b) Your answer to part (a) should involve the polyphase components of $h[n]$: $h_0[n] = h[3n]$, $h_1[n] = h[3n+1]$, $h_2[n] = h[3n+2]$. For the plots requested below, do all magnitude plots on one graph and you can do all phase plots on one graph.

(i) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot the magnitude of the DTFT of $h_0[n] = h[3n]$, $H_0(\omega)$, over $-\pi < \omega < \pi$.

(ii) For the general case where $h[n]$ is an arbitrary impulse response, express the DTFT of $h_1[n] = h[3n+1]$, denoted $H_1(\omega)$, in terms of $H(\omega)$.

(iii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot both the magnitude AND phase of the DTFT $h_1[n] = h[3n+1]$, $H_1(\omega)$, over $-\pi < \omega < \pi$.

(iv) For the general case where $h[n]$ is an arbitrary impulse response, express the DTFT of $h_2[n] = h[3n+2]$, denoted $H_2(\omega)$, in terms of $H(\omega)$.

(v) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot both the magnitude AND phase of the DTFT $h_2[n] = h[3n+2]$, $H_2(\omega)$, over $-\pi < \omega < \pi$.

(c) Consider that the input to the system in Figure 1 is a sampled version of the analog signal in Figure 2. For the remaining parts of this problem, the input signal is as defined below where $x_a(t)$ is the analog signal in Figure 2. Assume that $1/T_s = 1$ Hz is above the Nyquist rate for this signal. That is, even though this signal is not strictly bandlimited, assume that aliasing effects are negligible.

$$x[n] = x_a(nT_s), \quad T_s = 1 \ sec$$

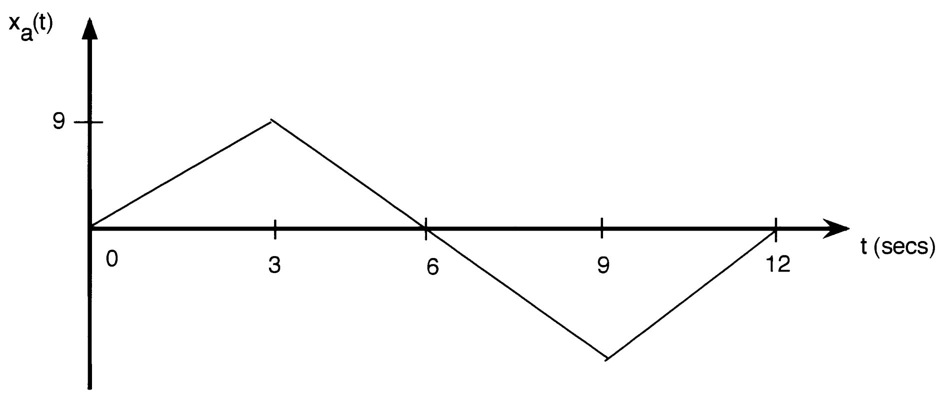


Figure 2.

(i) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine the output $y[n]$ of the system in Figure 1, when $x[n]$ is input to the system. Write output in sequence form (indicate where $n = 0$ is) OR do stem plot.

(ii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine the output $y_0[n] = x[n] * h_0[n]$, when $x[n]$ is input to the filter $h_0[n] = h[3n]$. Write output in sequence form (indicating where is $n = 0$) OR do stem plot.

(iii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine the output $y_1[n] = x[n] * h_1[n]$, when $x[n]$ is input to the filter $h_1[n] = h[3n+1]$. Write output in sequence form (indicating where $n = 0$ is) OR do stem plot.

(iv) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine the output $y_2[n] = x[n] * h_2[n]$, when $x[n]$ is input to the filter $h_2[n] = h[3n+2]$. Write output in sequence form (indicating where $n = 0$ is) OR do stem plot.

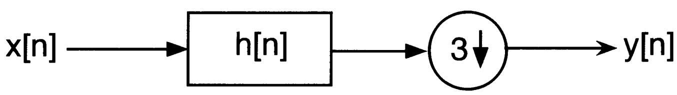
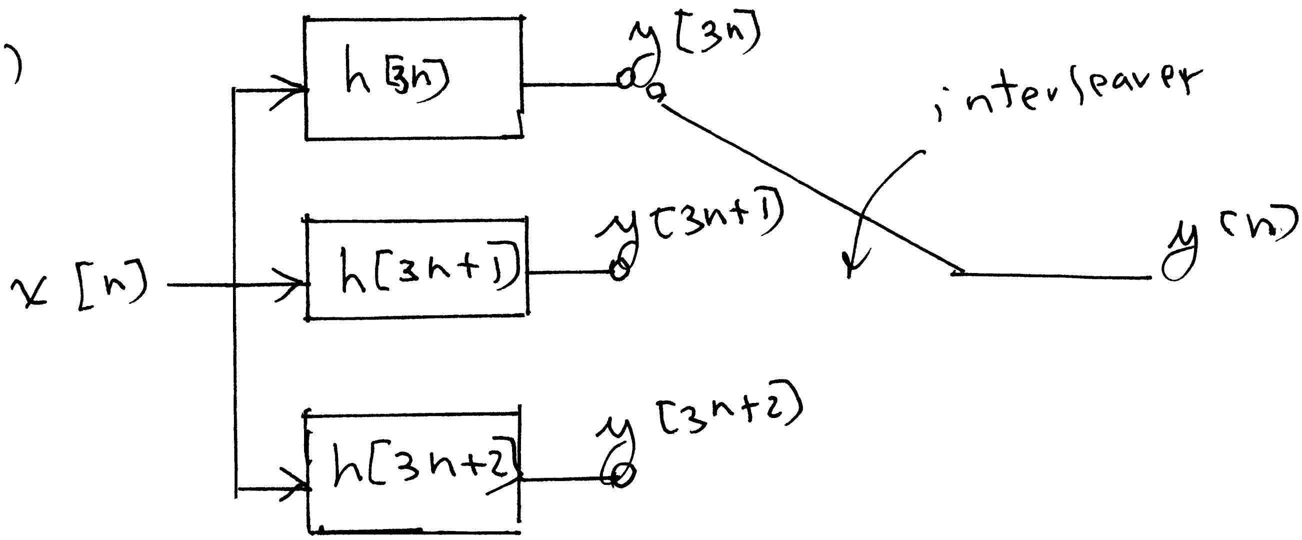


Figure 3.

(d) Draw a block diagram of an efficient implementation of the filtering followed by down-sampling system depicted in Fig. 3. Be sure to define all quantities.

(a)



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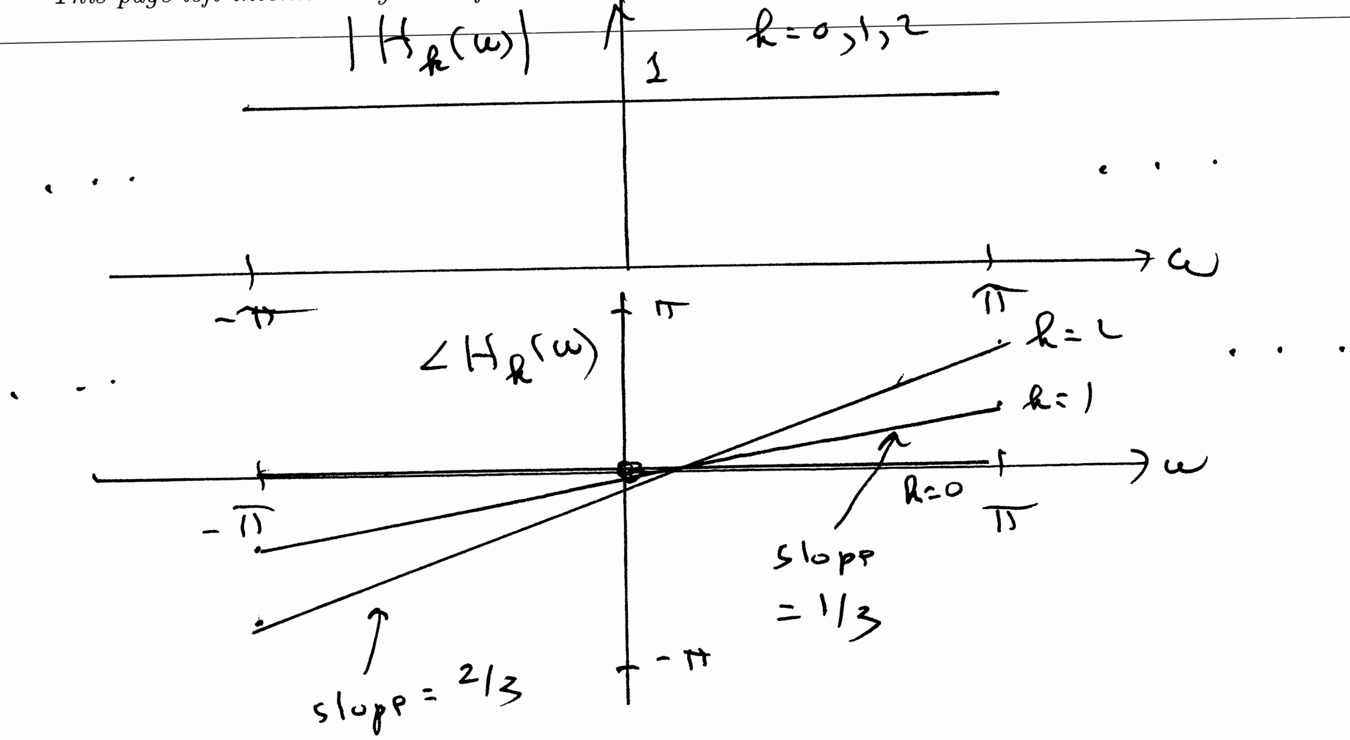
(b)



(b)-(ii)
(iv)

$$H_{\ell}(\omega) = \frac{1}{3}\sum_{k=0}^{2}\left\{e^{-j\frac{k2\pi\ell}{3}} H\left(\frac{\omega - k2\pi}{3}\right)\right\} e^{j\frac{\ell}{L}\omega} \quad \leftarrow L=3$$

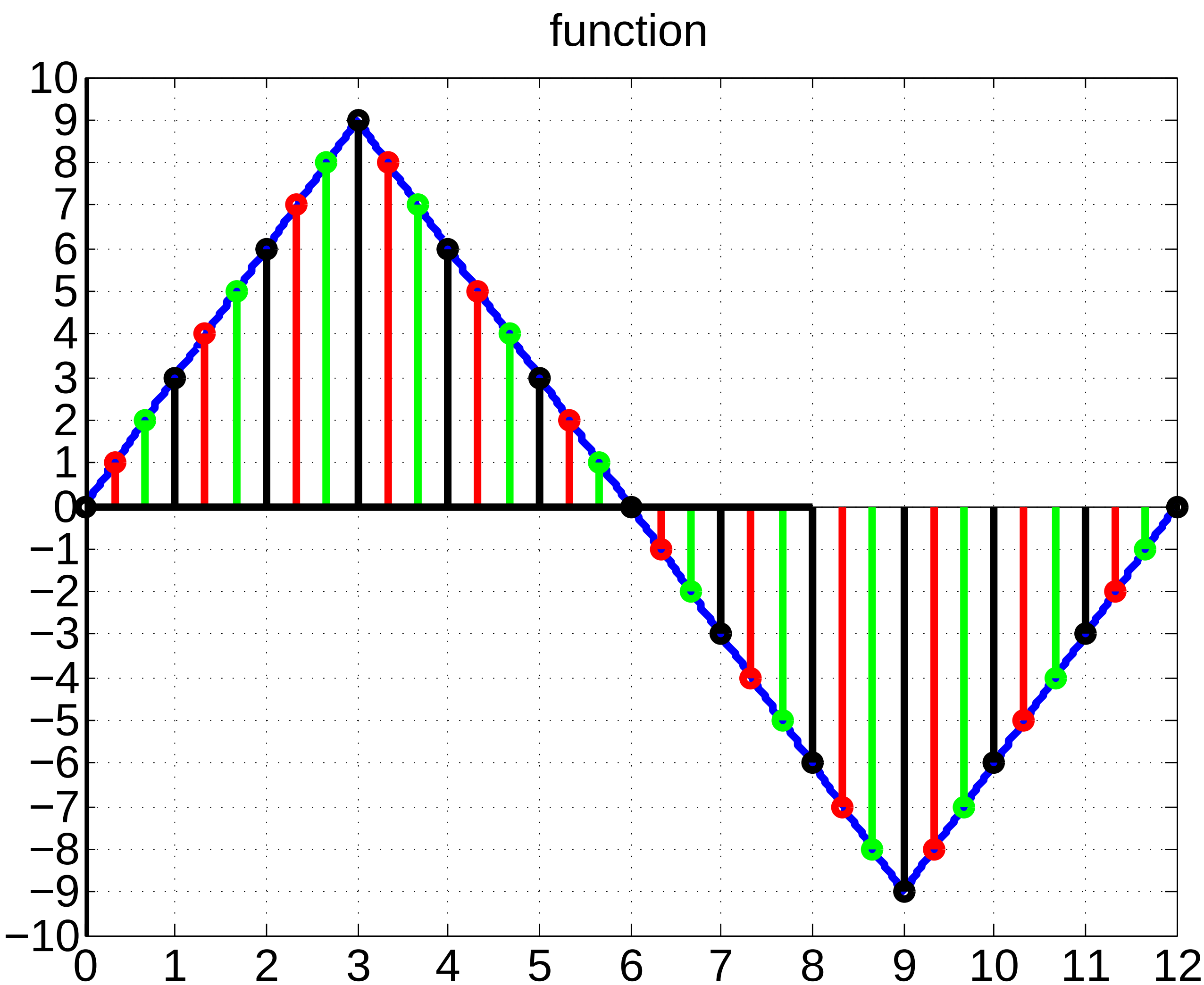
(c)-(i)

$y[n] = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, \ldots$
$-1, -2, -3, -4, -5, -6, -7, -8, -9, -8, -7, -6, -5, -4, \ldots$
$-3, -2, -1, 0\}$

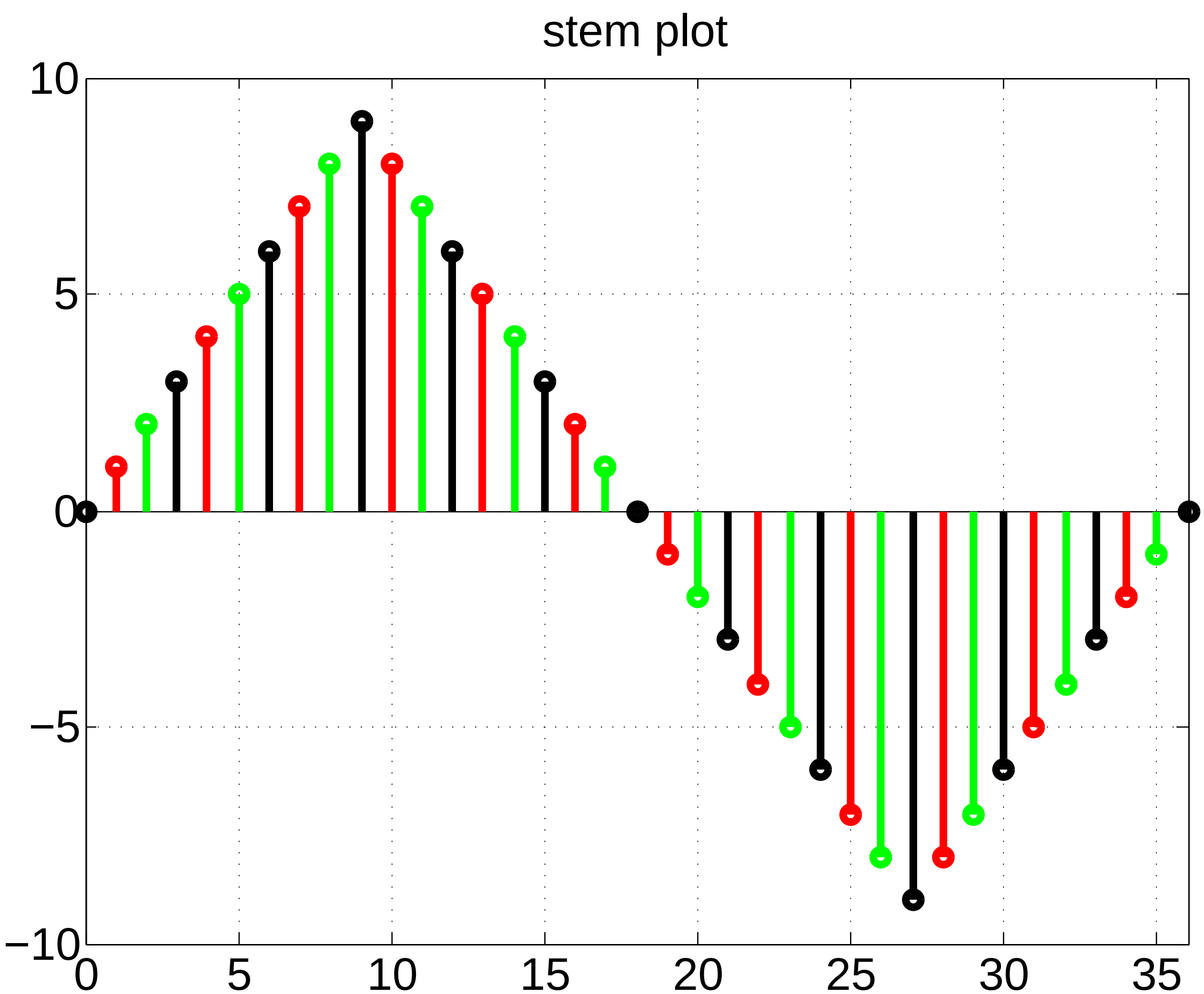
(c)-(ii) $y[3n] = \{0, 3, 6, 9, 6, 3, 0, -3, -6, -9, -6, -3, 0\}$

(c)-(iii) $y[3n+1] = \{1, 4, 7, 8, 5, 2, -1, -4, -7, -8, -5, -2, 0\}$

(c)-(iv) $y[3n+2] = \{2, 5, 8, 7, 4, 1, -2, -5, -8, -7, -4, -1\}$



function
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
0
1
2
3
4
5
6
7
8
9
10
11
12



stem plot
10
5
0
−5
−10
0
5
10
15
20
25
30
35

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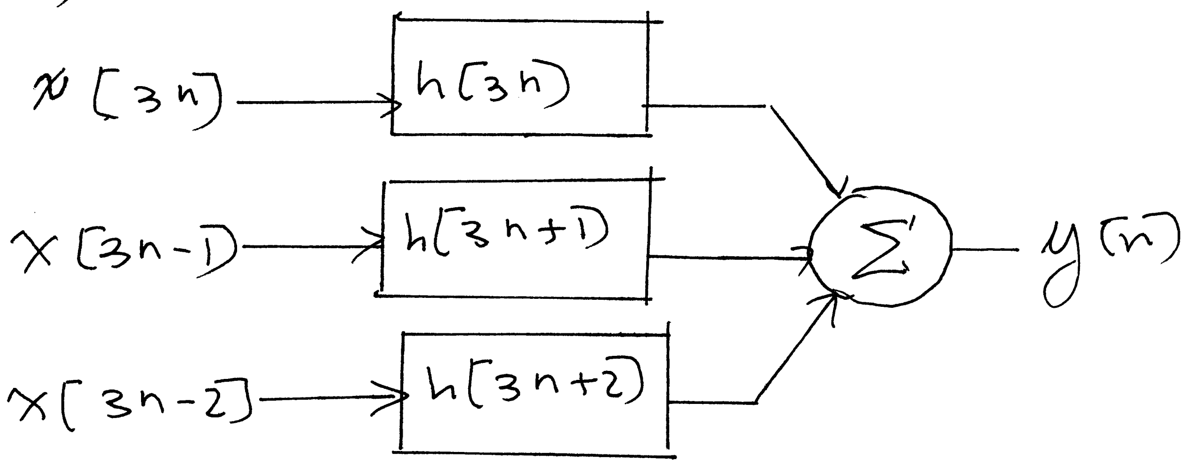
(d)



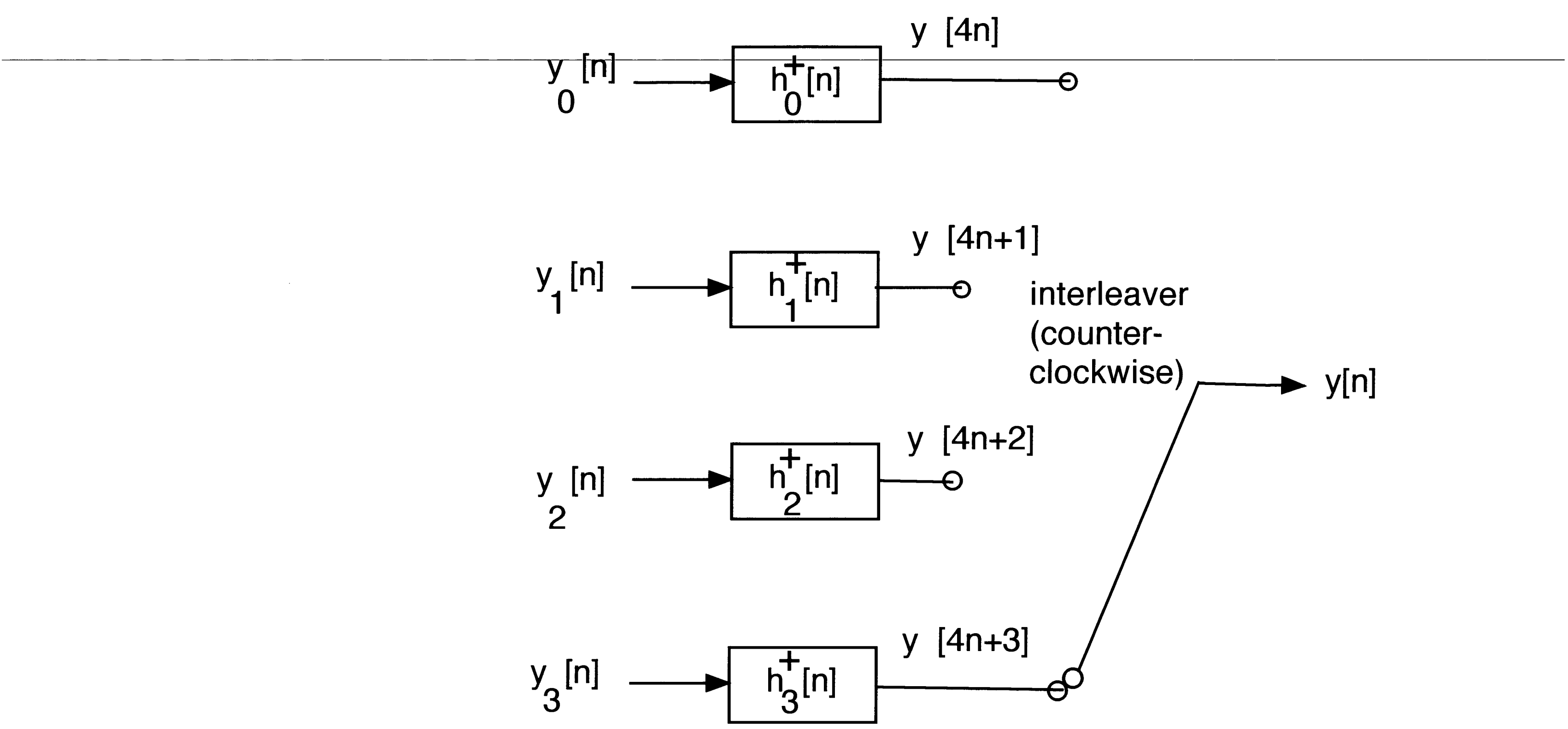


Figure 4.

Problem 2. This problem is about digital subbanding of four different DT signals. For the sake of simplicity, the signals are the four infinite-length sinewave signals defined below.

$$x_0[n] = \cos\left(\frac{\pi}{8}n\right) \quad x_1[n] = \cos\left(\frac{3\pi}{8}n\right) \quad x_2[n] = \cos\left(\frac{5\pi}{8}n\right) \quad x_3[n] = \cos\left(\frac{7\pi}{8}n\right)$$

Digital subbanding of these four signals is effected in an efficient way via the structure in Figure 4, where the various quantities are defined below: The impulse responses of the polyphase component filters are defined in terms of the ideal lowpass filter impulse response

$$h_{LP}[n] = 4\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \tag{1}$$

$$h_\ell^+[n] = h_{LP}[4n+\ell], \quad \ell = 0,1,2,3. \tag{2}$$

The respective signals at the inputs to these filters are formed from the input signals as described below, where $\hat{x}_k[n]$ is the Hilbert Transform of $x_k[n]$, k=0,1,2,3. **(a)** Plot the magnitude of the DTFT $Y(\omega)$ of the interleaved signal $y[n]$. Clearly indicate the frequencies of the sinewaves. **(b)** Draw a Block Diagram to recover the original signals, $x_k[n], k = 0,1,2,3$, for the general case (not just for sinewaves.) You can denote the cosine matrix in Eq (3) as **A** and the sine matrix in Eq (3) as **B**.

$$\begin{bmatrix} y_0[n] \\ y_1[n] \\ y_2[n] \\ y_3[n] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \cos\left(\frac{2\pi}{4}(1)\right) & \cos\left(\frac{4\pi}{4}(1)\right) & \cos\left(\frac{6\pi}{4}(1)\right) \\ 1 & \cos\left(\frac{2\pi}{4}(2)\right) & \cos\left(\frac{4\pi}{4}(2)\right) & \cos\left(\frac{6\pi}{4}(2)\right) \\ 1 & \cos\left(\frac{2\pi}{4}(3)\right) & \cos\left(\frac{4\pi}{4}(3)\right) & \cos\left(\frac{6\pi}{4}(3)\right) \end{bmatrix} \begin{bmatrix} x_0[n] \\ x_1[n] \\ x_2[n] \\ x_3[n] \end{bmatrix} \underset{\color{red}{-}}{+} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sin\left(\frac{2\pi}{4}(1)\right) & \sin\left(\frac{4\pi}{4}(1)\right) & \sin\left(\frac{6\pi}{4}(1)\right) \\ 0 & \sin\left(\frac{2\pi}{4}(2)\right) & \sin\left(\frac{4\pi}{4}(2)\right) & \sin\left(\frac{6\pi}{4}(2)\right) \\ 0 & \sin\left(\frac{2\pi}{4}(3)\right) & \sin\left(\frac{4\pi}{4}(3)\right) & \sin\left(\frac{6\pi}{4}(3)\right) \end{bmatrix} \begin{bmatrix} \hat{x}_0[n] \\ \hat{x}_1[n] \\ \hat{x}_2[n] \\ \hat{x}_3[n] \end{bmatrix} \tag{3}$$

my whole solution assumed there was a minus sign in the middle
OBVIOUSLY, I will accept answers for both cases:
minus sign or plus sign in the middle

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Each sinewave is up sampled by a factor of 4 $\Rightarrow$

frequency is divided by 4:

(i) $x_0^{up}[n] = \cos\left(\frac{\pi}{32} n\right)$

(ii) $x_1^{up}[n] = \cos\left(\frac{3\pi}{32} n\right)$ $\Rightarrow$ upper sideband starting at $\omega = \pi/2$

$\Rightarrow \frac{\pi}{2} + \frac{3\pi}{32} = \frac{19\pi}{32}$

with + sign in middle of Eqn (3), answer changes to lsb and freq= 13 π /32

(iii) $x_2^{up}[n] = \cos\left(\frac{5\pi}{32} n\right)$ $\Rightarrow$ lower sideband working backwards from π

$\Rightarrow \pi - \frac{5\pi}{32} = \frac{32\pi}{32} - \frac{5\pi}{32} = \frac{27\pi}{32}$

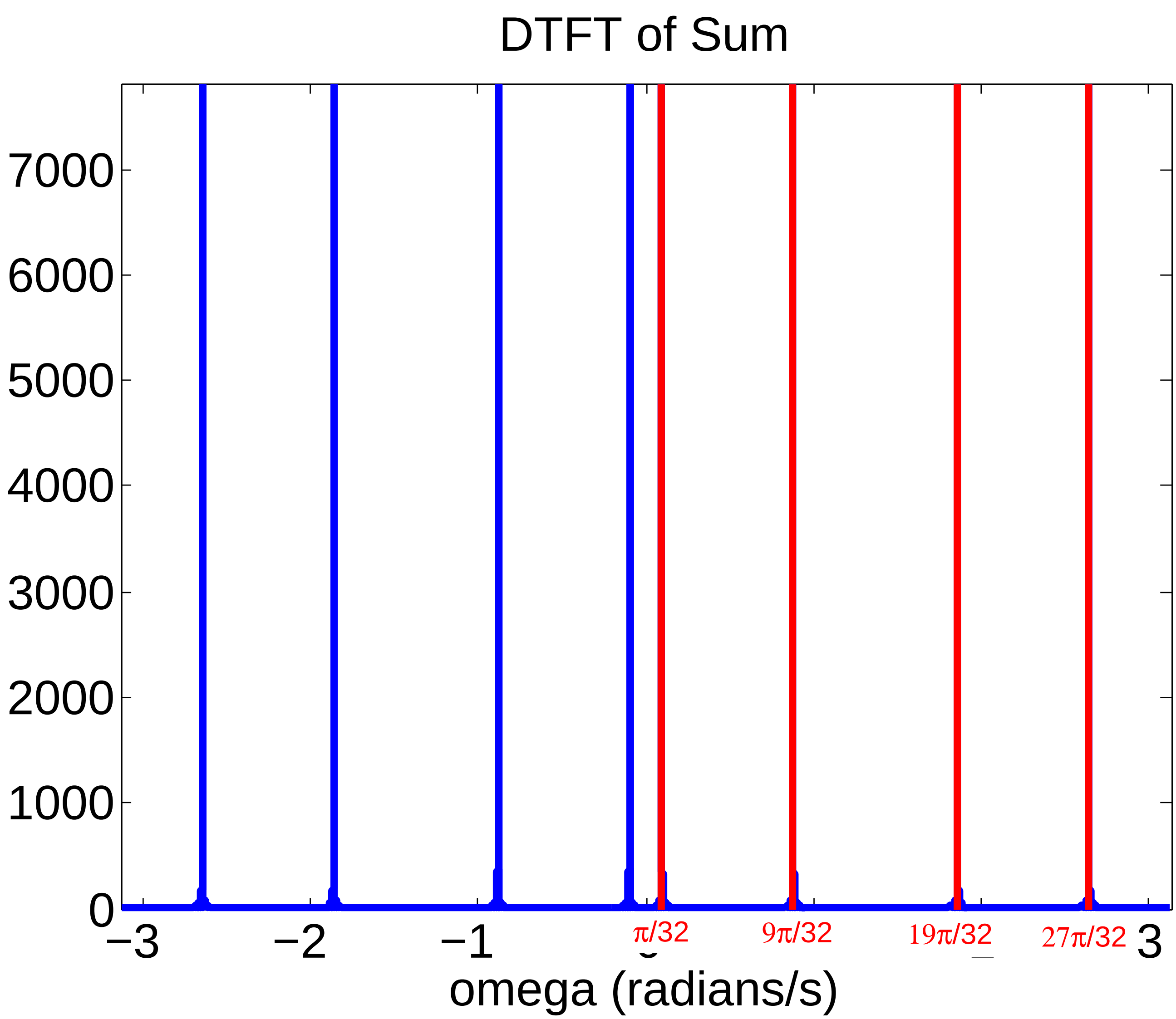
(iv) $x_3^{up}[n] = \cos\left(\frac{7\pi}{32} n\right)$ $\Rightarrow$ lower sideband working backwards from $\frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \frac{7\pi}{32} = \frac{9\pi}{32}$

with + sign in middle of Eqn (3), answer changes to usb and freq= 23 π /32

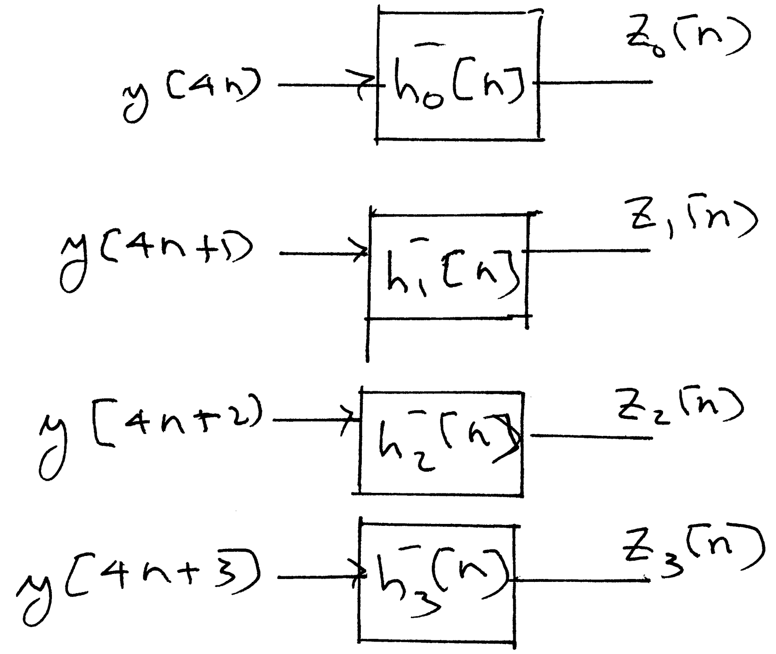
$\Rightarrow$ everything is real-valued, symmetric about $n=0$

$\Rightarrow$ negative frequencies are mirror image



DTFT of Sum
7000
6000
5000
4000
3000
2000
1000
0
−3
−2
−1
0
3
π/32
9π/32
19π/32
27π/32
omega (radians/s)

(b) block diagram



$$\begin{bmatrix} z_0[n) \\ z_1(n) \\ z_2(n] \\ z_3(n) \end{bmatrix} = A \begin{bmatrix} x_0(n) \\ x_1(n) \\ x_2(n) \\ x_3(n) \end{bmatrix} - B \begin{bmatrix} \hat{x}_0[n) \\ \hat{x}_1(n) \\ \hat{x}_2[n) \\ \hat{x}_3(n) \end{bmatrix}$$

$$A^T B = \underline{0}_{4\times 4}$$

$$A^T A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 4 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \qquad B^T B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

Thus:

$$\begin{bmatrix} u_0[n) \\ u_1[n) \\ u_2(n) \\ u_3(n) \end{bmatrix} = A^T z = \begin{bmatrix} 4x_0(n) \\ 2x_1(n]+2x_3(n) \\ 4x_2(n) \\ 2x_1(n)+2x_3(n] \end{bmatrix} ; \quad \underbrace{\begin{bmatrix} y_0(n] \\ y_1(n) \\ y_2(n) \\ y_3(n) \end{bmatrix}}_{\text{run each of these thru Hilbert Transformer to create } \hat{y}_k[n]} = B^T z = \begin{bmatrix} 0 \\ -2\hat{x}_1[n)+2\hat{x}_3[n] \\ 0 \\ +2\hat{x}_1(n)-2\hat{x}_3[n] \end{bmatrix}$$

$k=0,1,2,3$

$$\hat{y}_1[n] = +2x_1[n] - 2x_3[n]$$

$$\hat{y}_3[n] = -2x_1[n] + 2x_3[n]$$

Thus:

$$u_0[n] = 4x_0[n]$$

$$u_1[n] + \hat{v}_1[n] = 4x_1[n]$$

$$u_2[n] = 4x_2[n]$$

$$u_3[n] + \hat{v}_3[n] = 4x_3[n]$$

subscript three

My vector z is the same thing as the original y
The most compact way to write the final answer is:

$$x = A^T y + B^T \hat{y}$$

The plus sign + in the middle here would change to a minus sign - IF the minus sign - in the middle of Eqn 3 was changed to a plus sign -