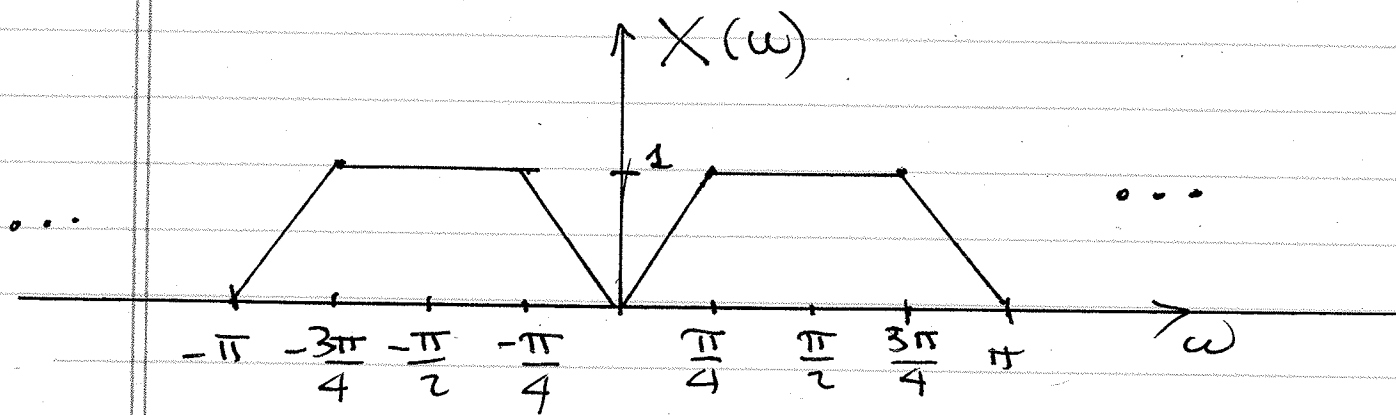


Problem 1 (a) $\frac{3\pi}{8} - \frac{\pi}{8} = \frac{2\pi}{8} = \frac{\pi}{4}$

$$\frac{3\pi}{8} + \frac{\pi}{8} = \frac{4\pi}{8} = \frac{\pi}{2}$$

• Multiplication by $\cos(\frac{\pi}{2}n)$ shifts to the left and right by $\frac{\pi}{2}$ in freq. domain



$$(b) \quad X_0(\omega) = X(\omega) H_0(\omega) \quad \left. \begin{array}{l} X_0[n] = \\ x[n] * h_0[n] \end{array} \right\}$$

$$X_0(\omega) = \frac{1}{2} X\left(\frac{\omega}{2}\right) H_0\left(\frac{\omega}{2}\right) + \frac{1}{2} X\left(\frac{\omega-2\pi}{2}\right)$$

As discussed in class
and developed in handout:

$$H_0\left(\frac{\omega-2\pi}{2}\right)$$

$$H_0\left(\frac{\omega-2\pi}{2}\right) = e^{j\frac{1}{2}\left(\frac{\omega-2\pi}{2}\right)}$$

$$= e^{j\frac{\omega}{4}} e^{-j\frac{\pi}{2}}$$

$$= -j e^{j\frac{\omega}{4}} \quad \text{for } |\omega| < \frac{\pi}{2}$$

$$H_0\left(\frac{\omega - 2\pi}{2}\right) = -j e^{j\frac{3\pi}{4}} \cos\left(\left|\frac{\omega - 2\pi}{2}\right| - \frac{\pi}{4}\right) \quad (1.2)$$

for $\frac{\pi}{2} < \omega < \frac{3\pi}{2}$

• Thus, there is aliasing over the frequency range $\frac{\pi}{2} < |\omega| < \pi$

• There is no aliasing over $0 < |\omega| < \frac{\pi}{2}$

• For $\frac{\pi}{2} < \omega < \pi$, the signal parts

$X\left(\frac{\omega}{2}\right)$ and $X\left(\frac{\omega - 2\pi}{2}\right)$ are both flat

(In fact, $X\left(\frac{\omega}{2}\right) = X\left(\frac{\omega - 2\pi}{2}\right)$ for the special case of this particular $x[n]$)

• Thus, over $\frac{\pi}{2} < \omega < \pi$, we obtain

$$X_0(\omega) =$$

$$\frac{1}{2} e^{j\frac{3\pi}{4}} \left\{ \cos\left(\frac{3}{2} - \frac{\pi}{4}\right) - j \cos\left(\left|\frac{\omega - 2\pi}{2}\right| - \frac{\pi}{4}\right) \right\}$$

$$= \frac{1}{2} e^{j\frac{3\pi}{4}} \left\{ \cos\left(\frac{3}{2} - \frac{\pi}{4}\right) - j \cos\left(\pi - \frac{3}{2} - \frac{\pi}{4}\right) \right\}$$

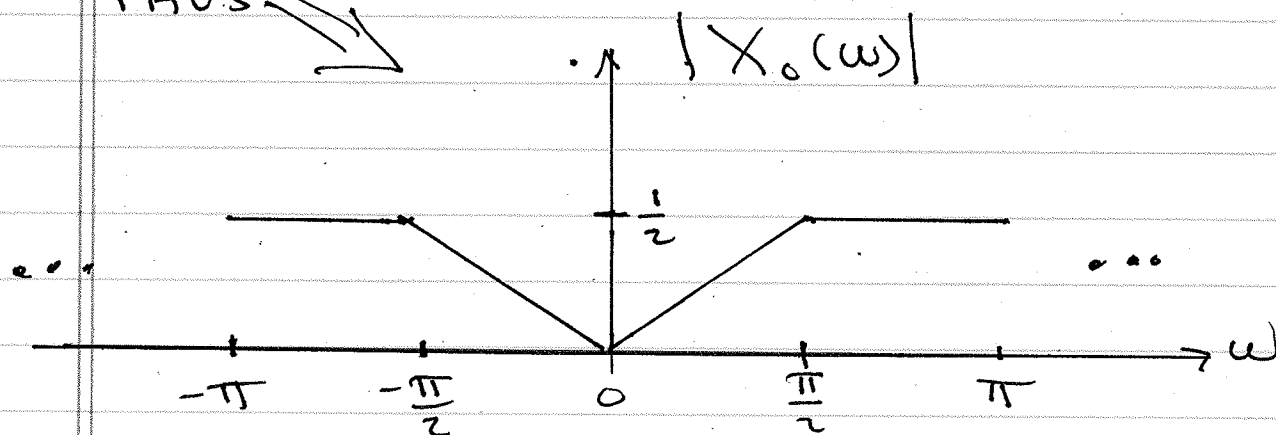
From handout called SRRC for QMF.pdf (1.3)
 we showed (for $\beta = 1/2$ in this case):

$$|X_0(\omega)|^2 = \frac{1}{4} \left\{ \cos^2\left(\frac{\omega}{2} - \frac{\pi}{4}\right) + \cos^2\left(\frac{3\pi}{4} - \frac{\omega}{2}\right) \right\}$$

$$= 1 \text{ for } \omega \text{ over } \frac{\pi}{2} < \omega < \pi$$

$\Rightarrow$ see next page

THUS $\Rightarrow$



part (c):

(*) Note first: } easier way to show $V_1(\omega) = V_0(\omega - \pi)$ on page 1.3a

For this $x[n]$, we have the property

$$x[n] = (-1)^n x[n] \quad \left. \vphantom{x[n]} \right\} \text{THUS: } X(\omega - \pi) = X(\omega)$$

Since: $x[n] = 0$ for n odd

Since: $\cos\left(\frac{\pi}{2}n\right) = 0$ for n odd

Now, recall

$$h_1[n] = (-1)^n h_0[n]$$

$$\begin{aligned} \text{Define: } v_1[n] &= x[n] * h_1[n] \\ &= ((-1)^n x[n]) * ((-1)^n h_0[n]) \end{aligned}$$

Appendix insert page $\Rightarrow$ development on next page

Examine below for $\frac{\pi}{2} < \omega < \pi$: 1,3a

$$\begin{aligned} & \cos^2\left(\frac{\omega}{2} - \frac{\pi}{4}\right) + \cos^2\left(\frac{3\pi}{4} - \frac{\omega}{2}\right) \\ &= \frac{1}{2} + \frac{1}{2} \cos\left(\omega - \frac{\pi}{2}\right) + \frac{1}{2} + \frac{1}{2} \cos\left(\frac{3\pi}{2} - \omega\right) \\ &= \frac{1}{2} + \frac{1}{2} \cos\left(\frac{\pi}{2} - \omega\right) + \frac{1}{2} + \frac{1}{2} \cos\left(\frac{3\pi}{2} - \frac{4\pi}{2} - \omega\right) \\ &= 1 + \frac{1}{2} \left\{ \cos\left(\frac{\pi}{2} - \omega\right) + \cos\left(-\frac{\pi}{2} - \omega\right) \right\} \\ &= 1 + \frac{1}{2} \left\{ \cos\left(\frac{\pi}{2} - \omega\right) + \cos\left(\frac{\pi}{2} + \omega\right) \right\} \end{aligned}$$

for $\frac{\pi}{2} < \omega < \pi$

these terms cancel each other

$$v_1[n] = x[n] * h_1[n] \xleftrightarrow{\text{DTFT}} V_1(\omega) = X(\omega) H_1(\omega)$$

$$V_1(\omega) = H_0(\omega - \pi) X(\omega)$$

$$= H_0(\omega - \pi) X(\omega - \pi)$$

$$= V_0(\omega - \pi)$$

since
 $X(\omega) = X(\omega - \pi)$

$$\text{where: } V_0(\omega) = H_0(\omega) X(\omega)$$

$$N_1[n] = \left((-1)^n x[n] \right) * \left((-1)^n h_0[n] \right) \quad (1.4)$$

$$= \sum_{k=-\infty}^{\infty} (-1)^k x[k] (-1)^{n-k} h_0[n-k]$$

$$= (-1)^n \sum_{k=-\infty}^{\infty} x[k] h_0[n-k]$$

$$= (-1)^n N_0[n] \quad \text{where: } N_0[n] = x[n] * h_0[n]$$

$$\text{THUS: } V_1(\omega) = V_0(\omega - \pi)$$

Now,

$$X_1(\omega) = \frac{1}{2} V_1\left(\frac{\omega}{2}\right) + \frac{1}{2} V_1\left(\frac{\omega - 2\pi}{2}\right)$$

$$= \frac{1}{2} V_0\left(\frac{\omega}{2} - \pi\right) + \frac{1}{2} V_0\left(\frac{\omega - 2\pi}{2} - \pi\right)$$

$$= \frac{1}{2} V_0\left(\frac{\omega - 2\pi}{2}\right) + \frac{1}{2} V_0\left(\frac{\omega}{2} - 2\pi\right)$$

$$= \frac{1}{2} V_0\left(\frac{\omega - 2\pi}{2}\right) + \frac{1}{2} V_0\left(\frac{\omega}{2}\right)$$

Since $V_0(\omega)$ is

periodic with period 2π

THUS:

$$X_1(\omega) = X_0(\omega)$$

for this particular problem

$$|X_1(\omega)| = |X_0(\omega)|$$

$$(d) \quad Y_0(\omega) = \frac{1}{2} H_0^2(\omega) X(\omega)$$

1.5

$$+ \frac{1}{2} H_0(\omega) H_0(\omega - \pi) X(\omega - \pi)$$

above is

from summary page for QMF published
posted at course web site

Now, again, for this particular signal $x[n]$

$$X(\omega - \pi) = X(\omega)$$

THUS:

$$Y_0(\omega) = \frac{1}{2} X(\omega) \left\{ H_0^2(\omega) + H_0(\omega) H_0(\omega - \pi) \right\}$$

- For $|\omega| < \frac{\pi}{4}$: $H_0^2(\omega) = e^{j\frac{\omega}{2}}$

and $H_0(\omega - \pi) = 0$

THUS: $|Y_0(\omega)| = |X(\omega)|$ for $|\omega| < \frac{\pi}{4}$

- For $\frac{3\pi}{4} < |\omega| < \pi$: $H_0(\omega) = 0$

THUS: $Y_0(\omega) = 0 \Rightarrow$ for $\frac{3\pi}{4} < |\omega| < \pi$

- For $\frac{\pi}{4} < \omega < \frac{3\pi}{4}$:

Recall: $H_0(\omega - \pi) =$

$$= e^{j\frac{(\omega - \pi)}{2}} \cos\left(|\omega - \pi| - \frac{\pi}{4}\right)$$

$$= -j e^{j\frac{\omega}{2}} \cos\left(\pi - \omega - \frac{\pi}{4}\right)$$

For $\frac{\pi}{4} < \omega < \frac{3\pi}{4}$:

1.6

$$\begin{aligned} H_0(\omega - \pi) &= -j e^{j\frac{3\pi}{2}} \cos\left(\frac{3\pi}{4} - \omega\right) \\ &= -j e^{j\frac{3\pi}{2}} \cos\left(\omega - \frac{3\pi}{4}\right) \end{aligned}$$

Examine: $H_0^2(\omega) + H_0(\omega) H_0(\omega - \pi)$

$$\begin{aligned} &= e^{j\frac{3\pi}{4}} \cos^2\left(\omega - \frac{\pi}{4}\right) \\ &\quad + \left(e^{j\frac{3\pi}{2}} \cos\left(\omega - \frac{\pi}{4}\right) \times (-j) e^{j\frac{3\pi}{2}} \cos\left(\omega - \frac{3\pi}{4}\right) \right) \\ &= e^{j\frac{3\pi}{4}} \cos\left(\omega - \frac{\pi}{4}\right) \left\{ \cos\left(\omega - \frac{\pi}{4}\right) - j \underbrace{\cos\left(\omega - \frac{\pi}{4} - \frac{\pi}{2}\right)}_{\cos\left(\omega - \frac{3\pi}{4}\right)} \right\} \\ &= e^{j\frac{3\pi}{4}} \cos\left(\omega - \frac{\pi}{4}\right) \left\{ \cos\left(\omega - \frac{\pi}{4}\right) - j \sin\left(\omega - \frac{\pi}{4}\right) \right\} \end{aligned}$$

$$\left| H_0^2(\omega) + H_0(\omega) H_0(\omega - \pi) \right|$$

$$= \left| \cos\left(\omega - \frac{\pi}{4}\right) \right| \sqrt{\cos^2\left(\omega - \frac{\pi}{4}\right) + \sin^2\left(\omega - \frac{\pi}{4}\right)}$$

$$= \cos\left(\omega - \frac{\pi}{4}\right) \quad \text{for} \quad \frac{\pi}{4} < \omega < \frac{3\pi}{4}$$

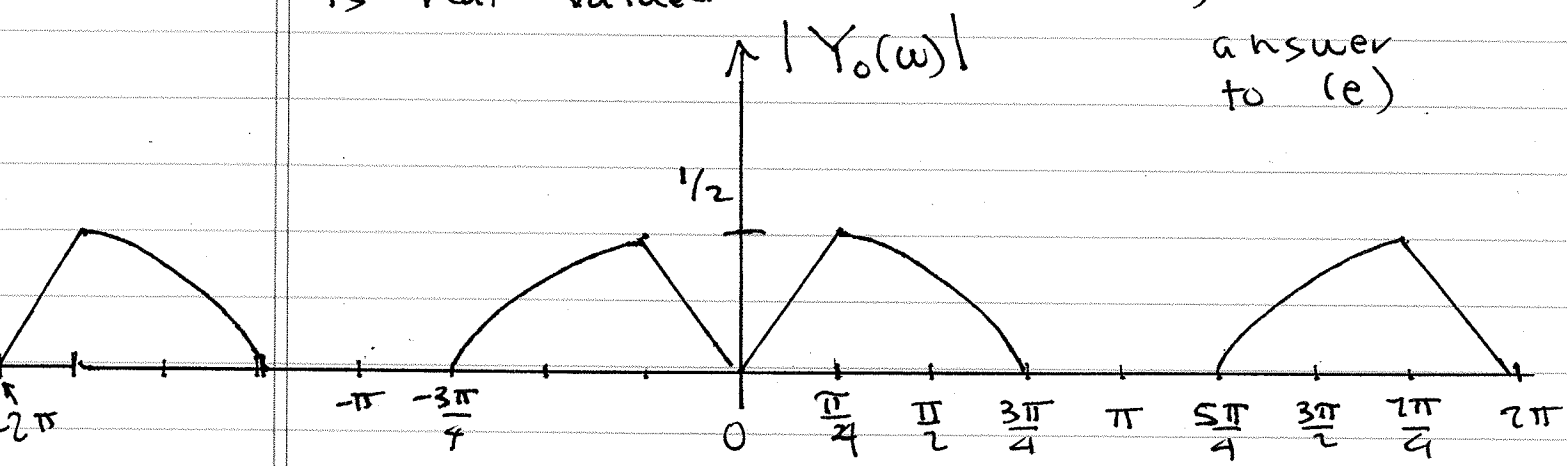
THUS:

$$|Y_0(\omega)| = \begin{cases} \frac{1}{2} |X_0(\omega)|, & \text{for } |\omega| < \frac{\pi}{4} \\ \frac{1}{2} \cos\left(\omega - \frac{\pi}{4}\right), & \text{for } \frac{\pi}{4} < \omega < \frac{3\pi}{4} \\ 0, & \text{for } \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$

Since $y_0[n]$ is real-valued,

$$|Y_0(-\omega)| = |Y_0(\omega)|$$

Note: every signal and filter in this problem is real-valued (in the time-domain)



(e)
part (e)

From QMF Summary diagram posted at course web site:

$$Y_1(\omega) = -\frac{1}{2} H_0^2(\omega - \pi) X(\omega) - \frac{1}{2} H_0(\omega) H_0(\omega - \pi) X(\omega - \pi)$$

Again, $X(\omega) = X(\omega - \pi)$ for this problem

$$|Y_1(\omega)| = \frac{1}{2} |X(\omega)| |H_0^2(\omega - \pi) + H_0(\omega) H_0(\omega - \pi)|$$

Previously for part (d), we examined

$$A(\omega) = H_0^2(\omega) + H_0(\omega) H_0(\omega - \pi)$$

Now, we are examining

$$B(\omega) = H_0^2(\omega - \pi) + H_0(\omega) H_0(\omega - \pi)$$

Note: $B(\omega) = A(\omega - \pi)$

$$= H_0^2(\omega - \pi) + H_0(\omega - \pi) H_0(\omega - 2\pi)$$

$$= H_0^2(\omega - \pi) + H_0(\omega - \pi) H_0(\omega)$$

THUS:

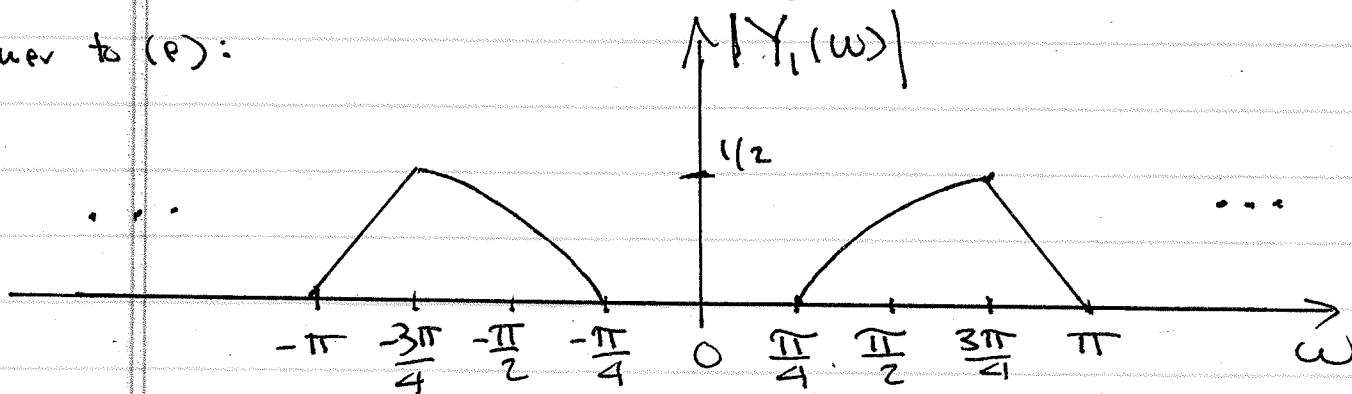
$$|Y_1(\omega)| = \frac{1}{2} |X(\omega)| |A(\omega - \pi)|$$

$$= \frac{1}{2} |X(\omega - \pi)| |A(\omega - \pi)|$$

for this problem

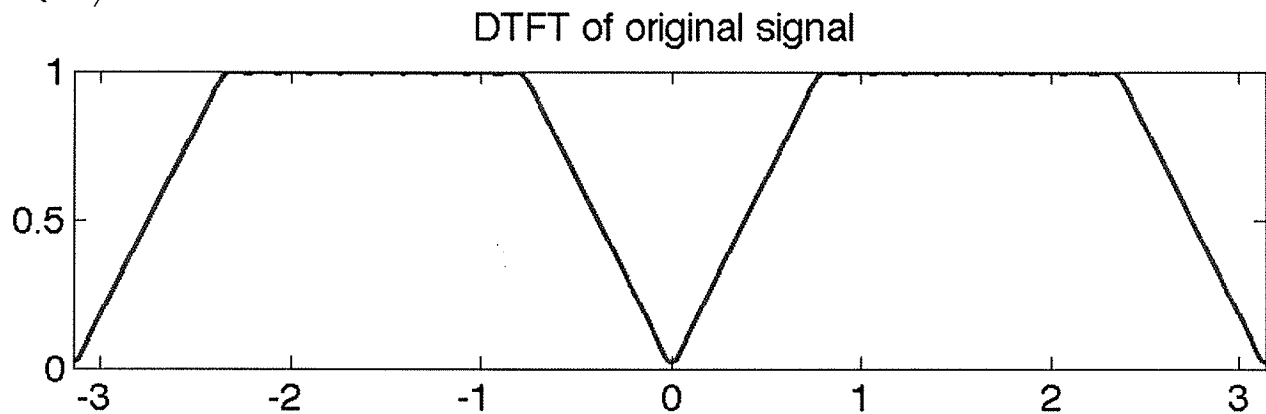
$\Rightarrow$ just shift answer to (d) to right by π } that's why I drew two periods of $|Y_0(\omega)|$

answer to (e):

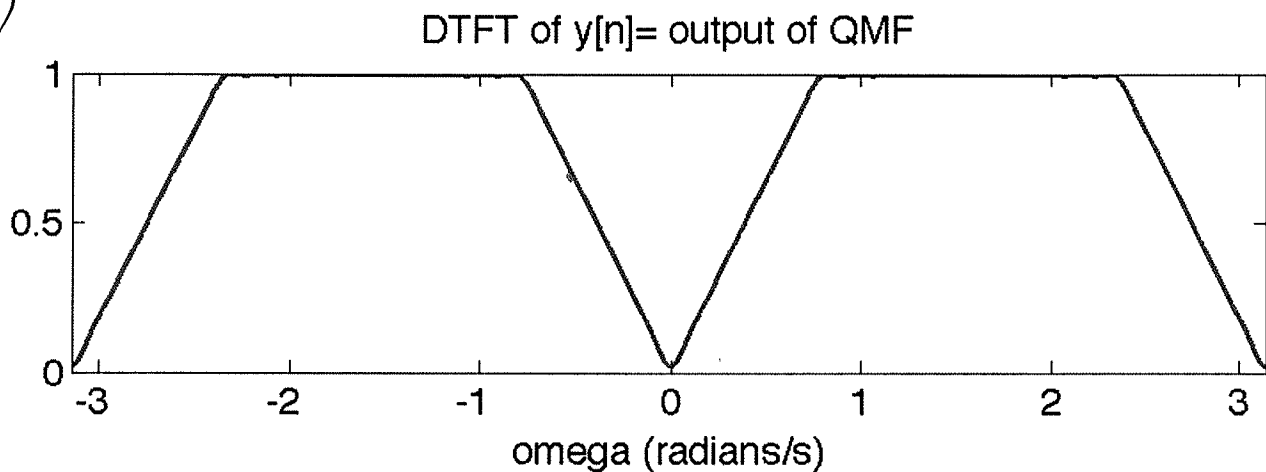


Plots generated by actual implementation in matlab. The magnitude of the DTFT is plotted in all cases.

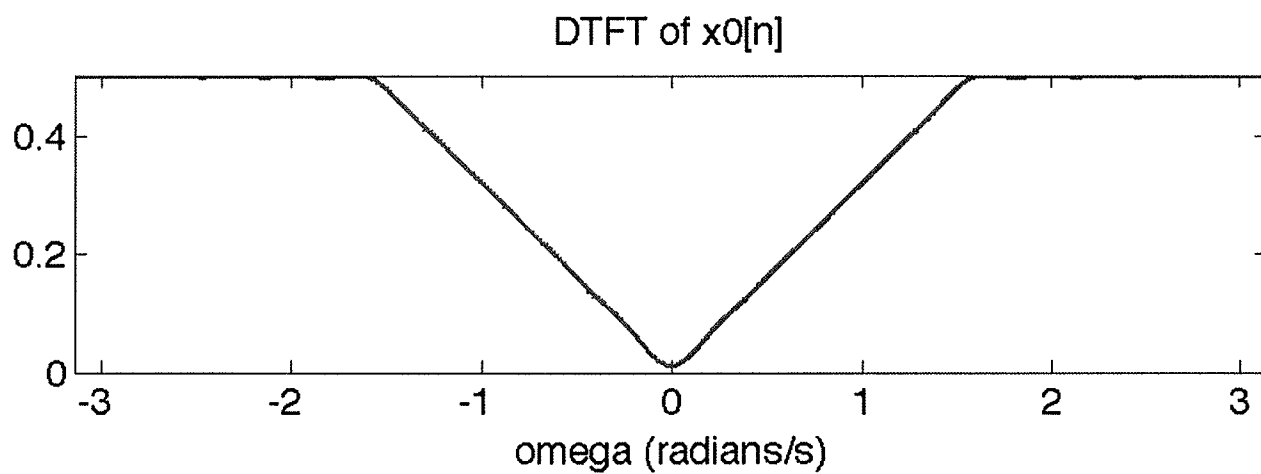
(a)



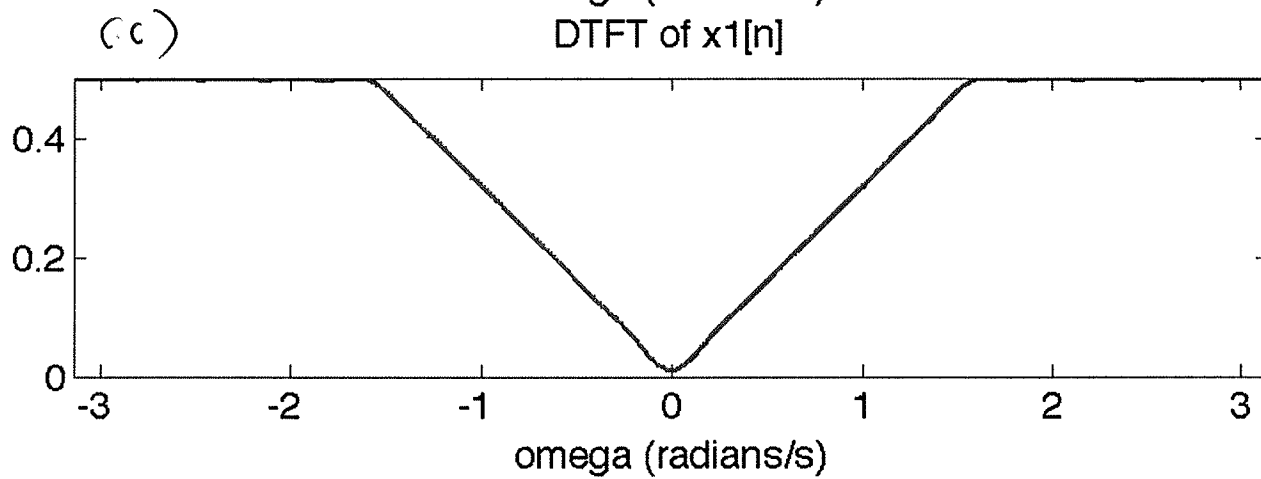
(f)



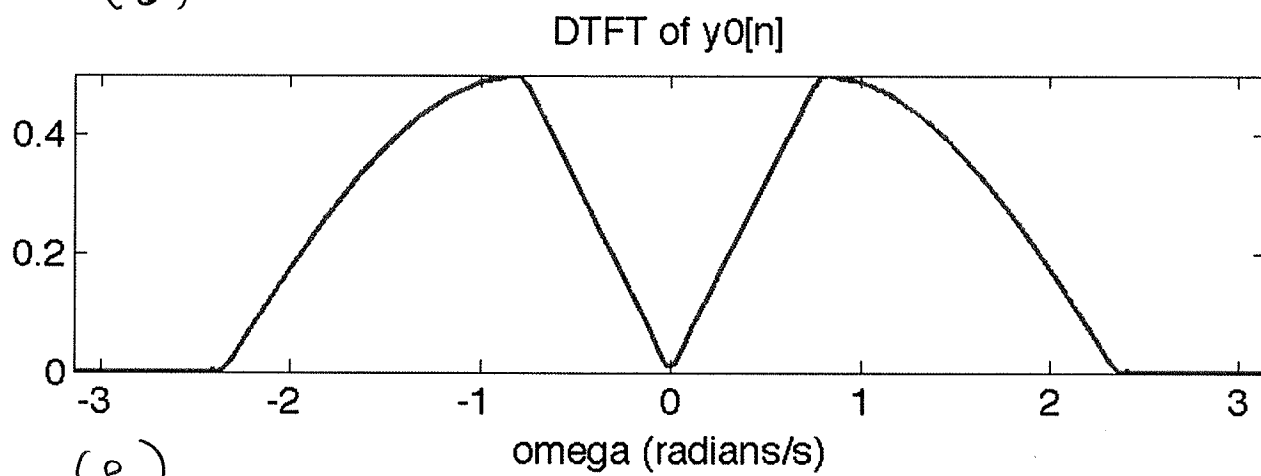
(b)



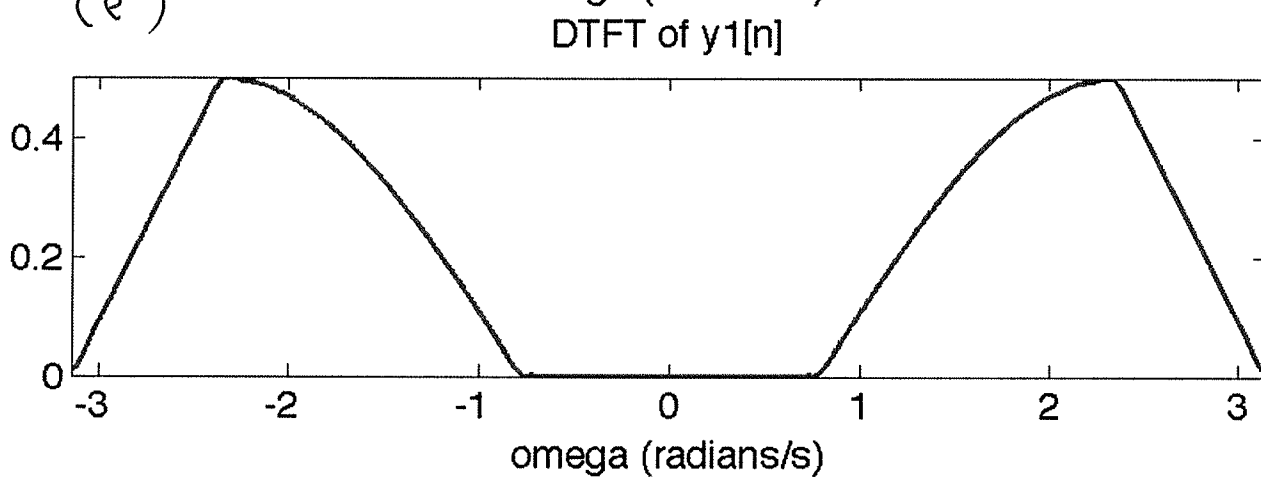
(c)



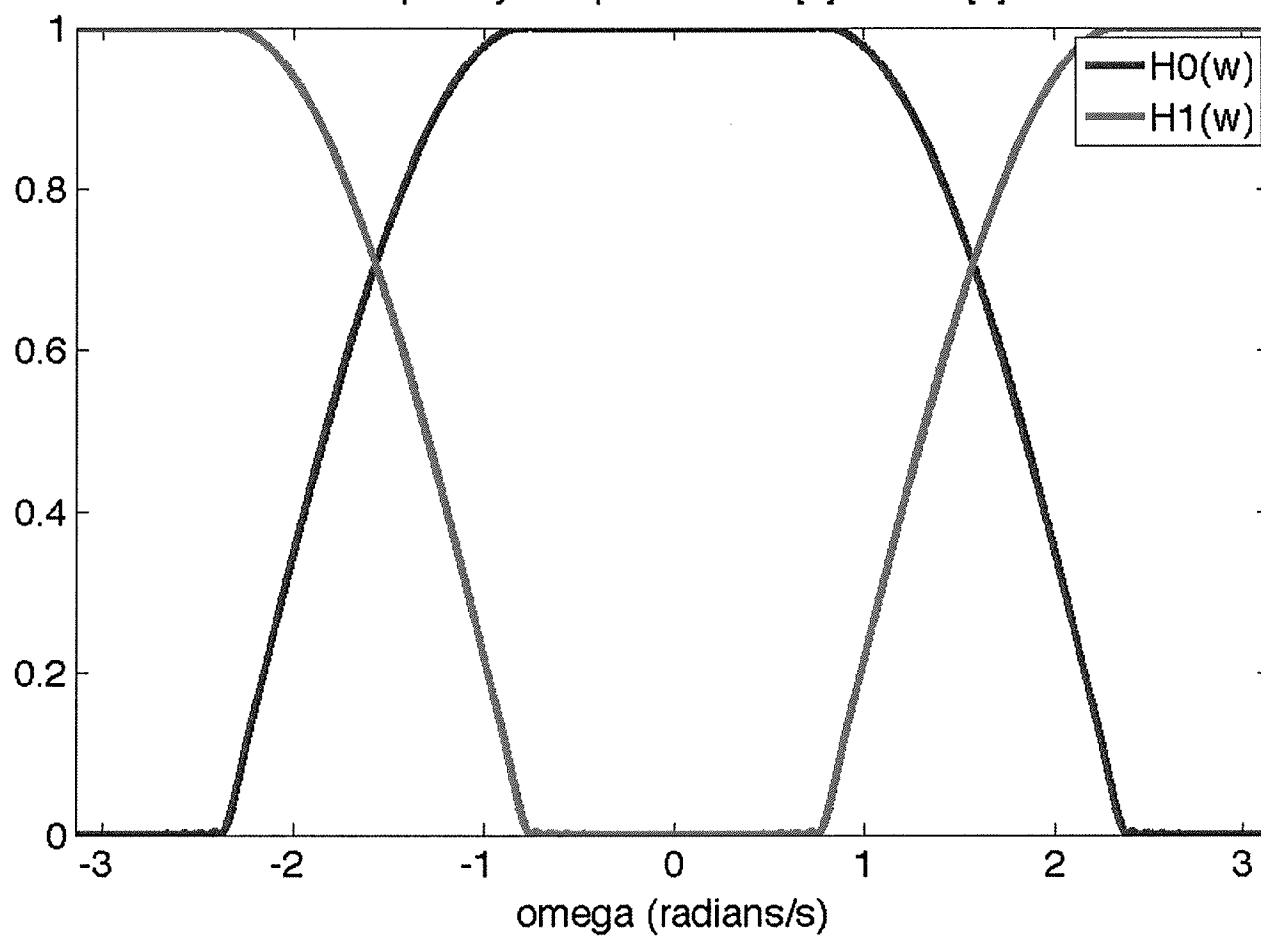
(d)



(e)



Frequency Response of $h_0[n]$ and $h_1[n]$

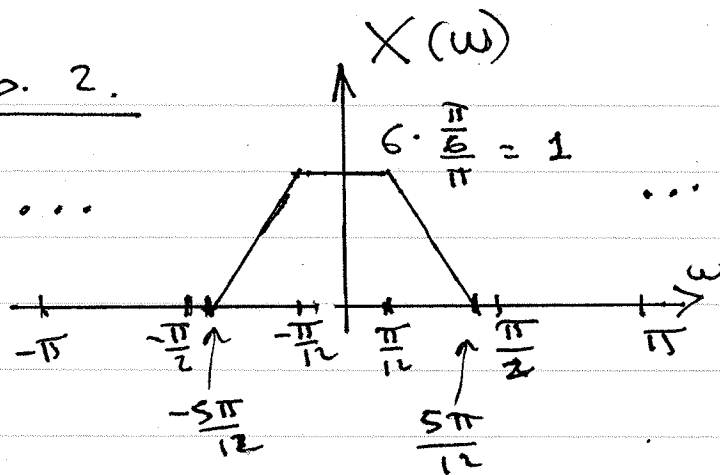


ECE 538 Exam 2 Solutions Fall 2010

(2.1)

Prob. 2.

Part (b): ...



$$\frac{\pi}{4} - \frac{\pi}{6} =$$

$$= \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$$

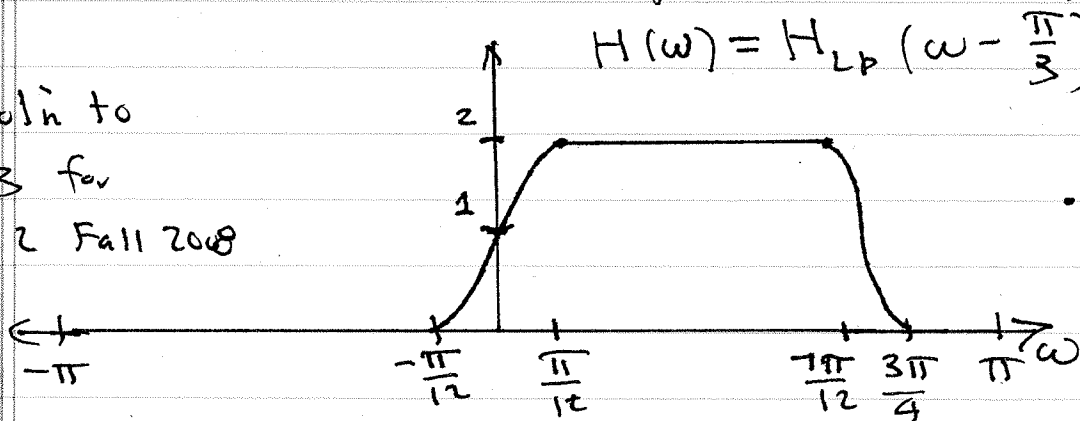
$$\frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$$

$$X[n] = 6 \underbrace{\sin\left(\frac{\pi}{4}n\right)}_{\pi n} \underbrace{\sin\left(\frac{\pi}{6}n\right)}_{\pi n}$$

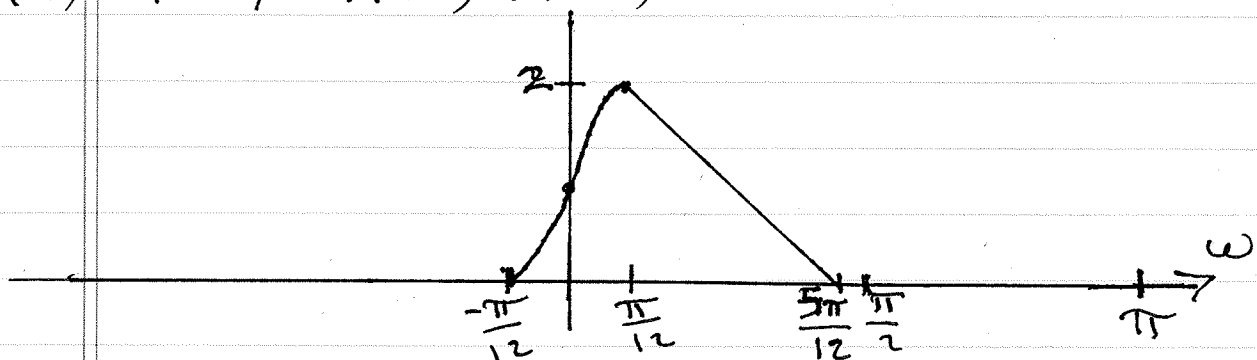
changed
from 4

Part (a) Same filter as in previous exams: Exam 2 Fall 2008 Problem 3

See Soln to
Prob. 3 for
Exam 2 Fall 2008



$$(c) Y(\omega) = H(\omega) X(\omega)$$



Prob. 2 (d)

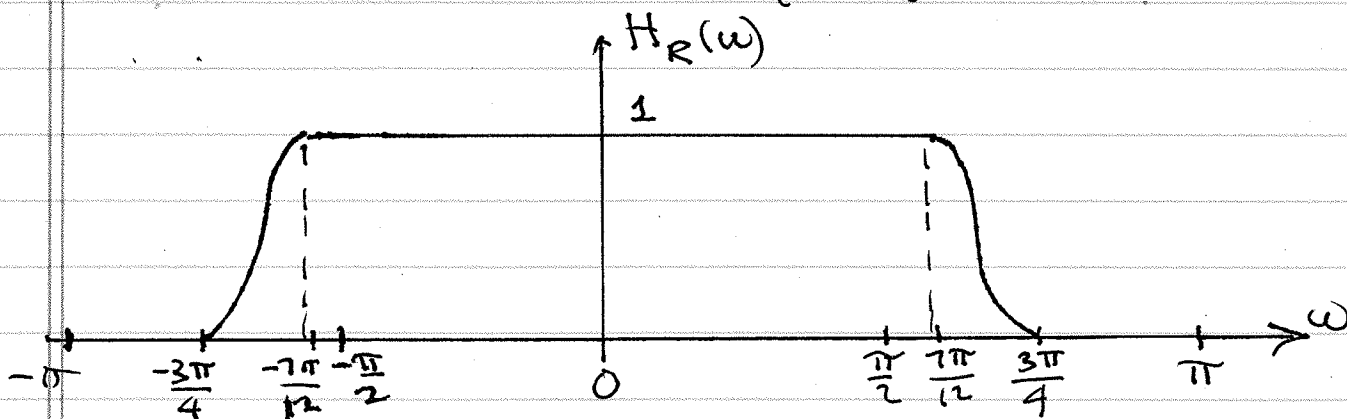
$$h_R[n] = h_{LP}[n] \cos\left(\frac{\pi}{3}n\right)$$

2.2

$$H_R(\omega) = \frac{1}{2} H_{LP}\left(\omega - \frac{\pi}{3}\right) + \frac{1}{2} H_{LP}\left(\omega + \frac{\pi}{3}\right)$$

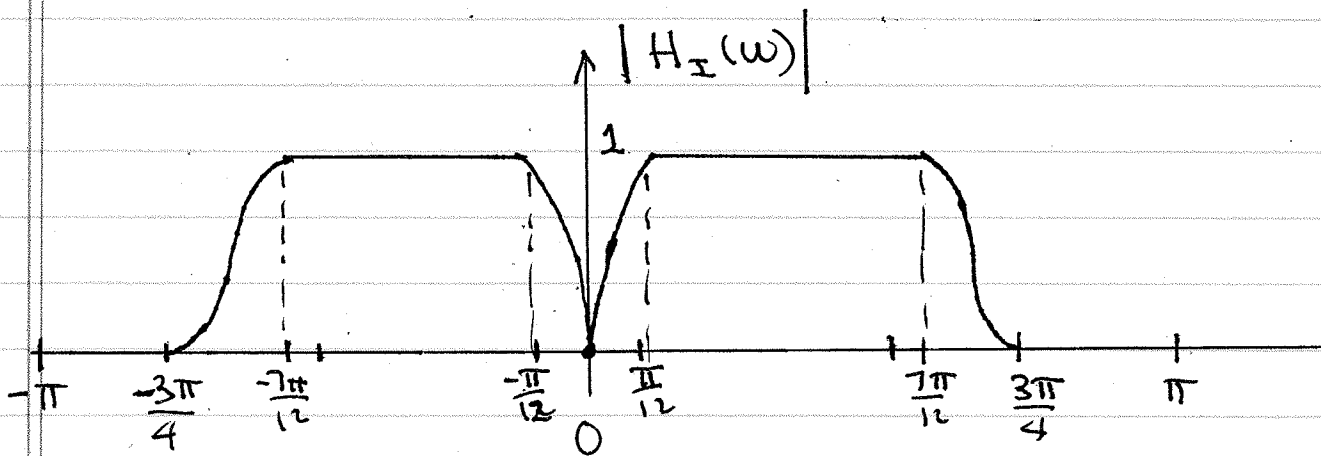
OR: $H_R(\omega) = \frac{1}{2} H(\omega) + \frac{1}{2} H^*(-\omega)$

Since: $h_R[n] = \text{Re}\{h[n]\} = \frac{1}{2} h[n] + \frac{1}{2} h^*[n]$



(e) $H_I(\omega) = \frac{1}{2j} H_{LP}\left(\omega - \frac{\pi}{3}\right) - \frac{1}{2j} H_{LP}\left(\omega + \frac{\pi}{3}\right)$

OR: $H_I(\omega) = \frac{1}{2j} H(\omega) - \frac{1}{2j} H^*(-\omega)$



(f) VSB is designed so that

(2.3)

$$y_R[n] = \operatorname{Re}\{y[n]\} = x[n]$$

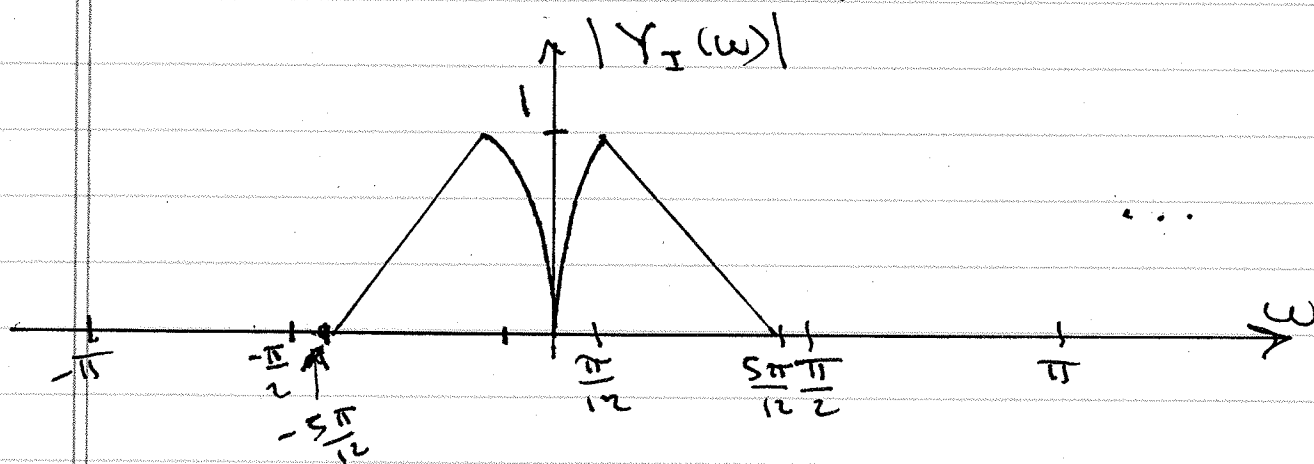
Since $H_R(\omega) = 1$ for $|\omega| < \frac{5\pi}{12}$

$$Y_R(\omega) = X(\omega)$$

$\Rightarrow$ same plot as part (b) answer

$$(g) Y_I(\omega) = X(\omega) H_I(\omega)$$

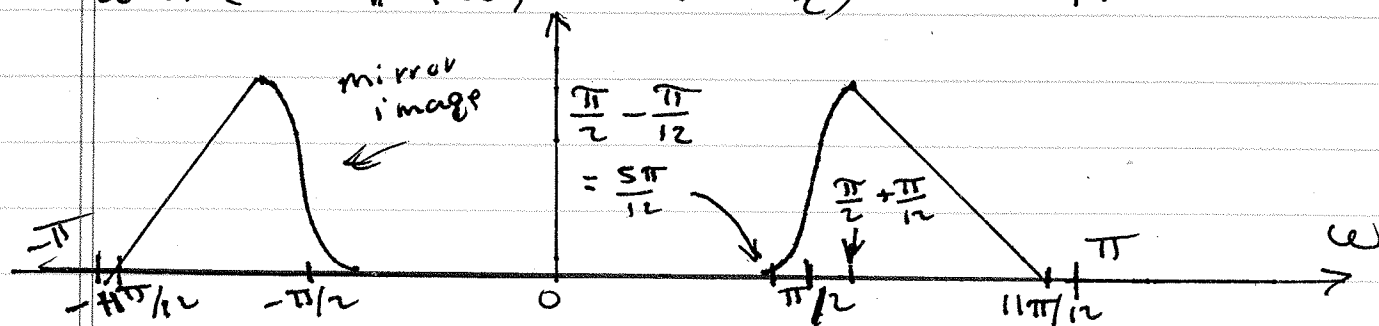
$$|Y_I(\omega)| = |X(\omega)| |H_I(\omega)|$$



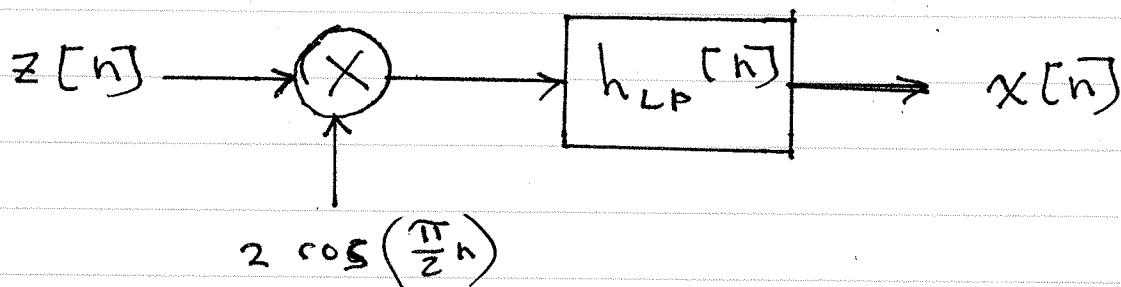
(h) Hint was given: $z[n] = \operatorname{Re}\{y[n]e^{j\frac{\pi}{2}n}\}$

Thus: $Z(\omega) = \frac{1}{2} V(\omega) + \frac{1}{2} V^*(-\omega)$

where: $V(\omega) = Y(\omega - \frac{\pi}{2}) \Rightarrow$ upper sideband



Prob. 2 (i)



Since: $z[n] = 2 \cos\left(\frac{\pi}{2}n\right)$

$$= y_R[n] 2 \cos^2\left(\frac{\pi}{2}n\right) - y_I[n] 2 \sin\left(\frac{\pi}{2}n\right) \cos\left(\frac{\pi}{2}n\right)$$

$$= y_R[n] + \underbrace{y_R[n] \cos(\pi n) - y_I[n] \sin(\pi n)}_{\text{filtered out by LPF}}$$

$$= x[n] \quad \text{since } y_R[n] = x[n]$$

Specs on LPF:

passband edge: $\omega_p = \frac{5\pi}{12}$

stopband edge: $\pi - \frac{5\pi}{12} = \frac{7\pi}{12}$