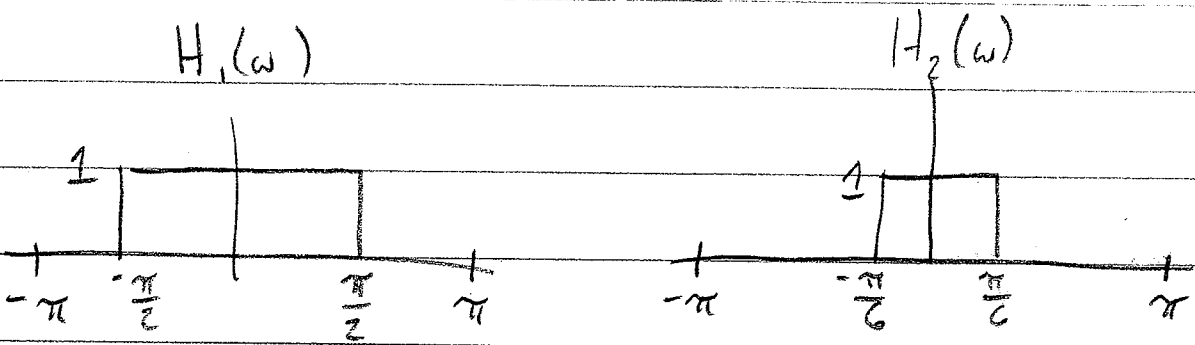


Prob 1

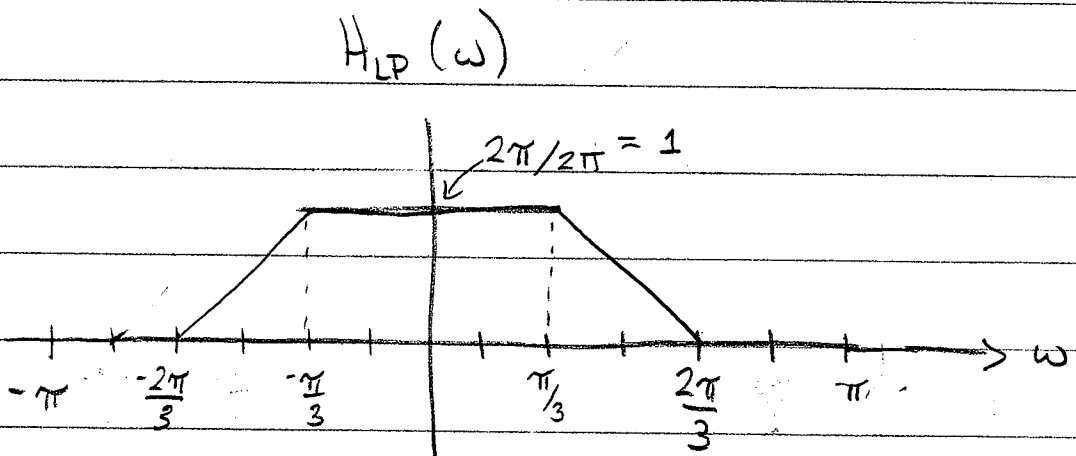
(a) $h_{LP}[n] = G h_1[n] h_2[n]$

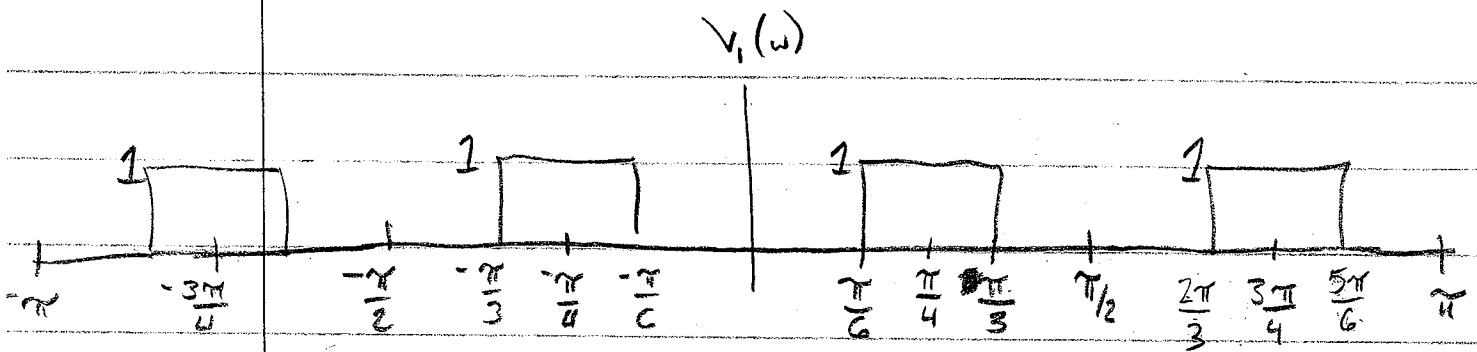
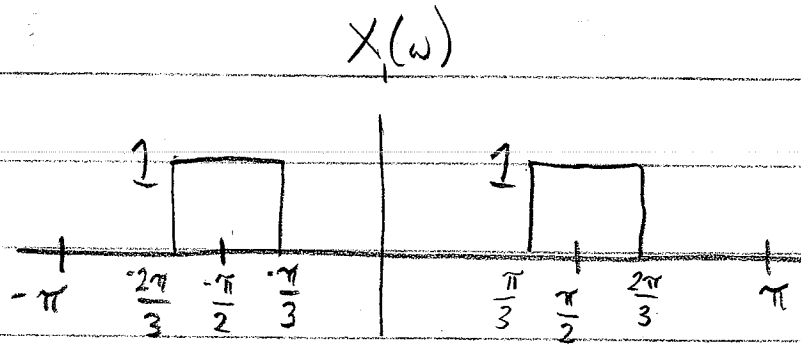
$h_1[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$

$h_2[n] = \frac{\sin\left(\frac{\pi}{6}n\right)}{\pi n}$



$H(\omega) = \frac{1}{2\pi} G (H_1 * H_2)(\omega)$

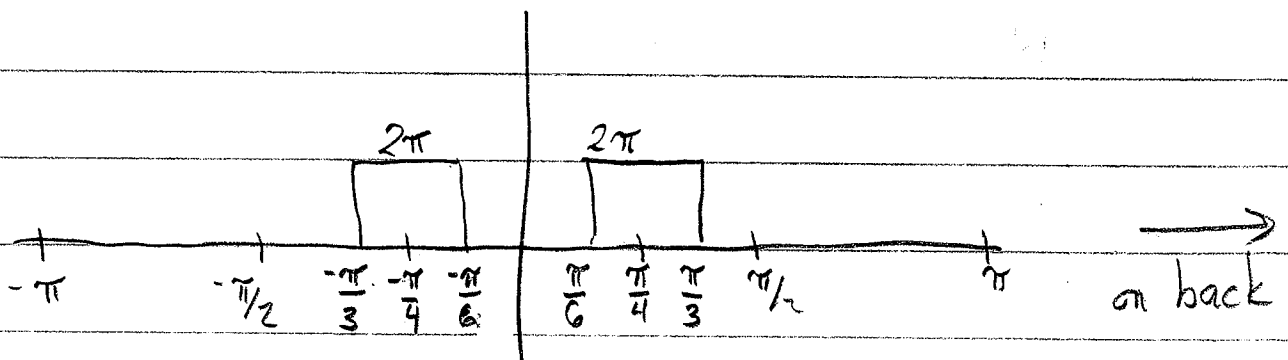




"Don't Care" region does not have any effect on upsampling since the upsampling by 2 ensures all info from $x[n]$ lies within $-\pi/3 < \omega < \pi/3$, which is inside "don't care" regions.

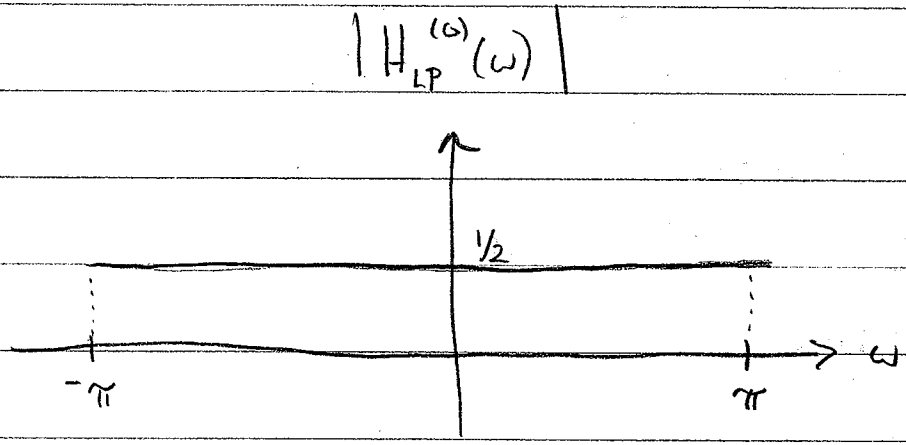
(b)

$$Y_1(\omega) = H_{LP}(\omega) V_1(\omega)$$



$$\begin{aligned}
 (c) \quad h_{LP}^{(o)}[n] &= h_{LP}[2n] = G \frac{\sin \pi n}{\pi 2n} \cdot \frac{\sin \frac{\pi}{3} n}{\pi 2n} \quad \forall n \\
 &= \frac{G}{4} \frac{\sin \pi n}{\pi n} \frac{\sin \frac{\pi}{3} n}{\pi n} \quad \forall n \\
 &= \frac{3G}{2} \delta[n] \cdot \frac{1}{3}
 \end{aligned}$$

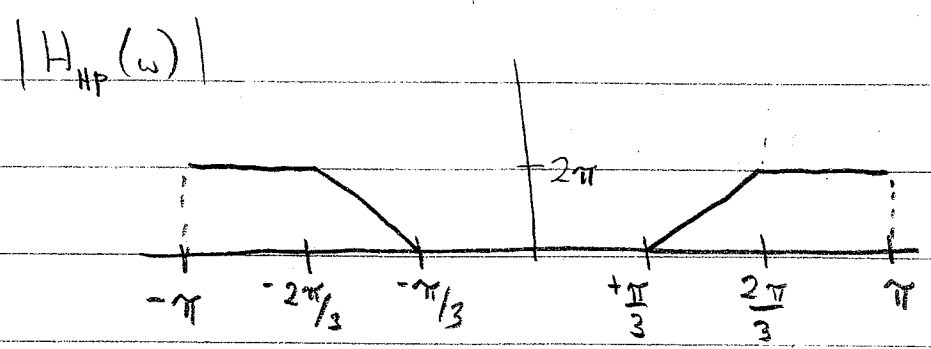
(i) $h_{LP}^{(o)}[n] = \frac{1}{2} \delta[n]$



(ii) No. $y_1^{(o)}[n] = x_1[n] * \frac{1}{2} \delta[n] = \frac{1}{2} x_1[n]$

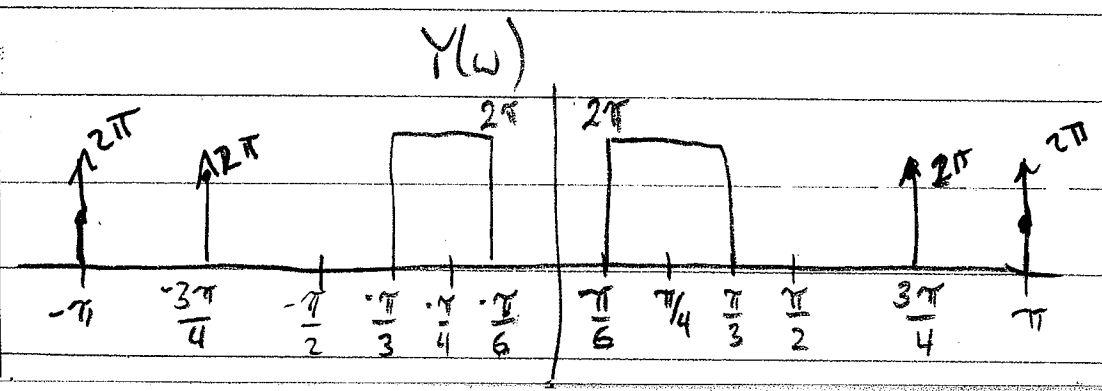
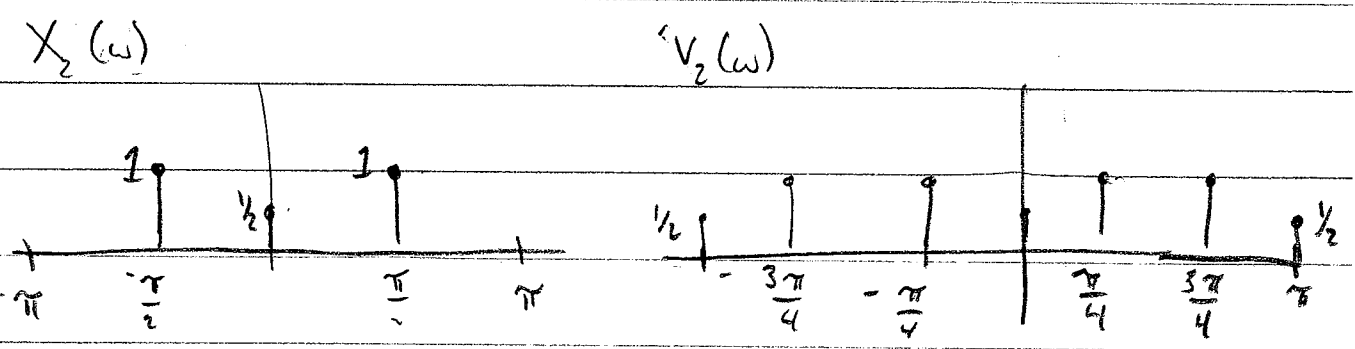
Prob 2

(a) $h_{HP}[n] = (-1)^n h_{LP}[n] = (e^{-j\pi})^n h_{LP}[n]$

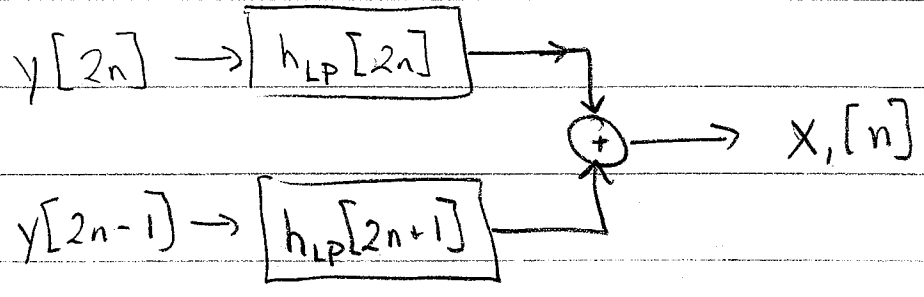


$h_{HP}^{(6)}[n] = (-1)^{2n} \frac{6}{4} \frac{\sin(\pi n)}{\pi n} \frac{\sin(\frac{\pi}{3} n)}{\pi n} = \frac{1}{2} \delta[n]$

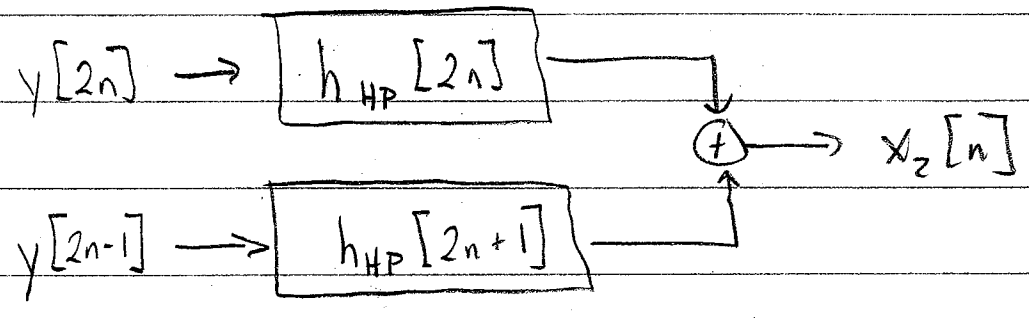
$x_2[n] = 1 + e^{-j\frac{\pi n}{2}} + e^{j\frac{\pi n}{2}} = \cos(2\pi n) + 2 \cos(\frac{\pi}{2} n)$



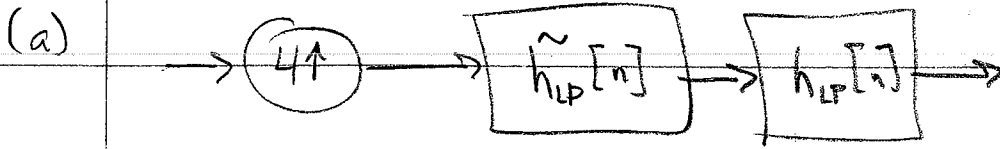
(b)



(c)

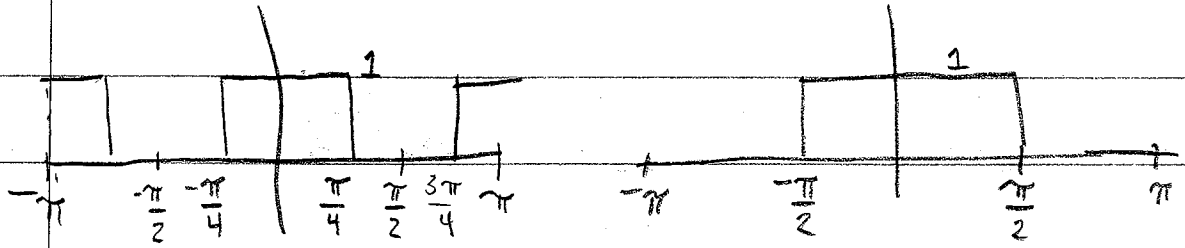


Prob 3

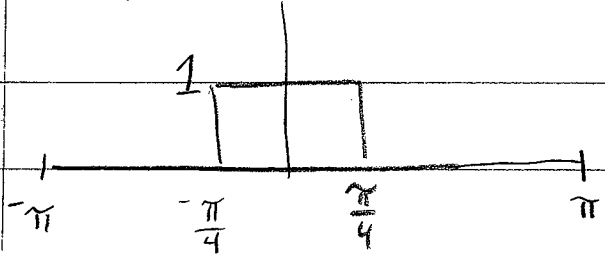


$$\tilde{H}'_{LP}(\omega) = H_{LP}(2\omega)$$

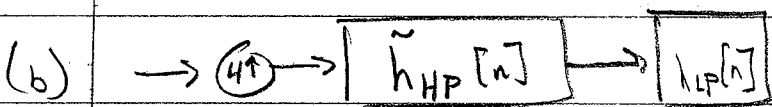
$$H_{LP}(\omega)$$



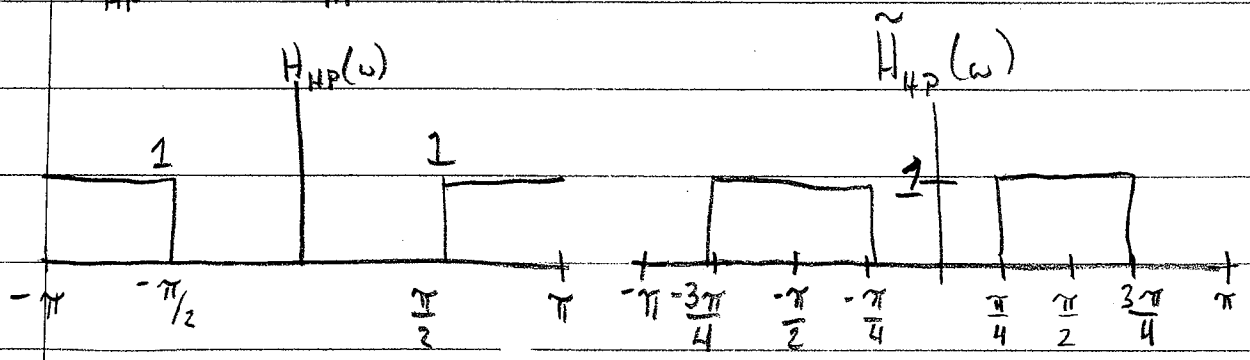
$$H_a(\omega)$$

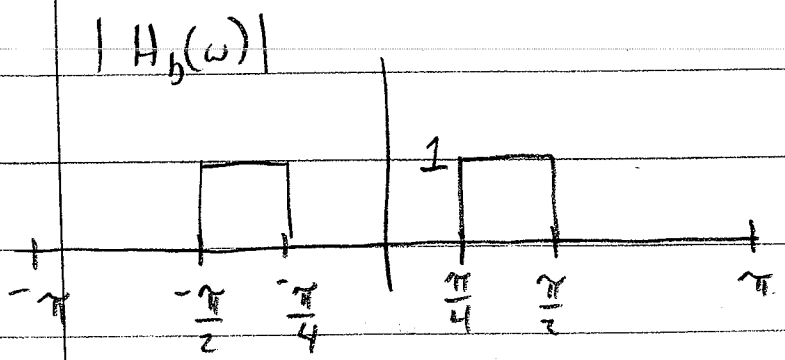


$$\Rightarrow h_a[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}$$



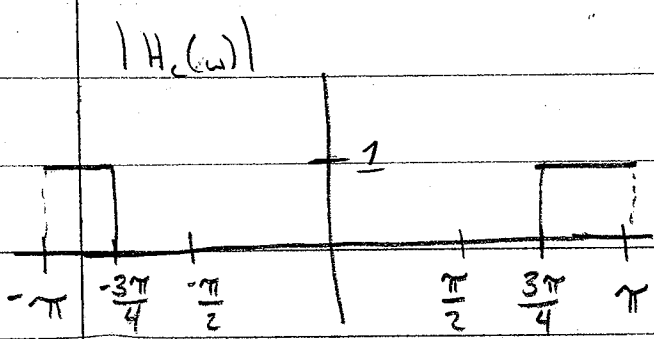
$$\tilde{H}_{HP}(\omega) = H_{HP}(2\omega)$$





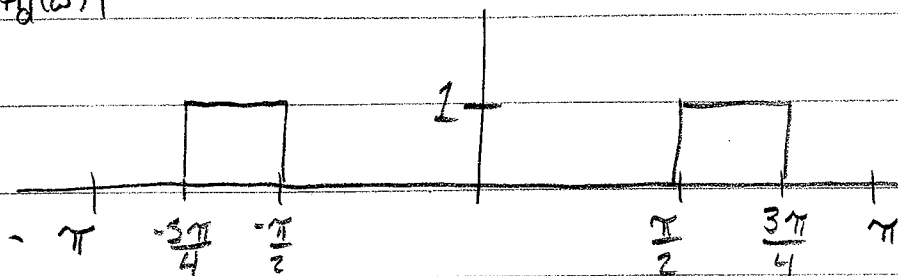
$$h_b[n] = \frac{2 \sin\left(\frac{\pi}{8}n\right) \cos\left(\frac{3\pi}{8}n\right)}{\pi n}$$

(c) $H_c(\omega) = H_{LP}(\omega) H_{HP}(\omega)$



$$h_c[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} e^{j\pi n}$$

$$(d) H_d(\omega) = \tilde{H}_{HP}(\omega) H_{HP}(\omega)$$

 $|H_d(\omega)|$


$$h_d[n] = \frac{2 \sin\left(\frac{\pi}{8}n\right) \cos\left(\frac{5\pi}{8}n\right)}{\pi n}$$