$x[n] = x_a\left(\frac{n}{F_s}\right)$ $\qquad x_a(t) \overset{\mathcal{F}}{\longleftrightarrow} X_a(f)$

$X_a(f) = 0$ for $|f| > W$

$F_s = 8W$

$x[n] \overset{DTFT}{\longleftrightarrow} X(w)$

Within $-\pi < w < \pi$, what region of $X(w) = 0$??

Find where max frequency, W, is mapped to thru

$$w = 2\pi \frac{f}{F_s}$$

$$w_{max} = 2\pi \frac{W}{8W} = \frac{2\pi}{8} = \frac{\pi}{4}$$

ANS: $X(w) = 0$ for $\frac{\pi}{4} < |w| < \pi$

Note: $\frac{\sin(\pi n)}{\pi n} = \delta[n] \overset{\text{DTFT}}{\longleftrightarrow} 1 \quad \forall \omega$

So a student thinks

$$\frac{\sin\left(\pi\left(n+\frac{3}{4}\right)\right)}{\pi\left(n+\frac{3}{4}\right)} \overset{\text{DTFT}}{\longleftrightarrow} \underbrace{e^{j\frac{3}{4}\omega}}$$

is this correct $\forall \omega$? for all ω?

No! 1. $e^{j\frac{3}{4}\omega}$ is not periodic with period 2π

$$e^{j\frac{3}{4}(\omega+2\pi)} = e^{j\frac{3}{4}\omega} e^{j\frac{6\pi}{4}} = e^{j\frac{3}{4}\omega}(-j) \neq e^{j\frac{3}{4}\omega}$$

The period is 8π

2. $e^{j\frac{3}{4}\omega}$ does not account for the discontinuity in phase that occurs at $\omega = \pm\pi$ when the linear phase has a fractional slope

CORRECT ANSWER: See the VIP_MultirateFormulas.pdf

$y[n] = x[2n] \overset{\text{DTFT}}{\underset{??}{\longleftrightarrow}} \frac{1}{2} X\left(\frac{\omega}{2}\right)$ } is this correct?

No! $X\left(\frac{\omega}{2}\right)$ is periodic with period 4π

BUT a DTFT has to be periodic w/ period 2π

ALSO: $\frac{1}{2} X(\omega/2)$ does not account for any aliasing that might have occurred by lowering the sampling rate by a factor of 2.

Correct answer is in VIP_MultivateFormulas.pdf

$$Y(\omega) = \frac{1}{2} X\left(\frac{\omega}{2}\right) + \frac{1}{2} X\left(\frac{\omega - 2\pi}{2}\right)$$

If $x[n] = x_a(nT_s)$,

$$y[n] = x[n] * \frac{\sin\left(\pi\left(n+\frac{3}{4}\right)\right)}{\pi\left(n+\frac{3}{4}\right)} = ??$$

ANS: $y[n] = x_a(nT_s + .75T_s)$

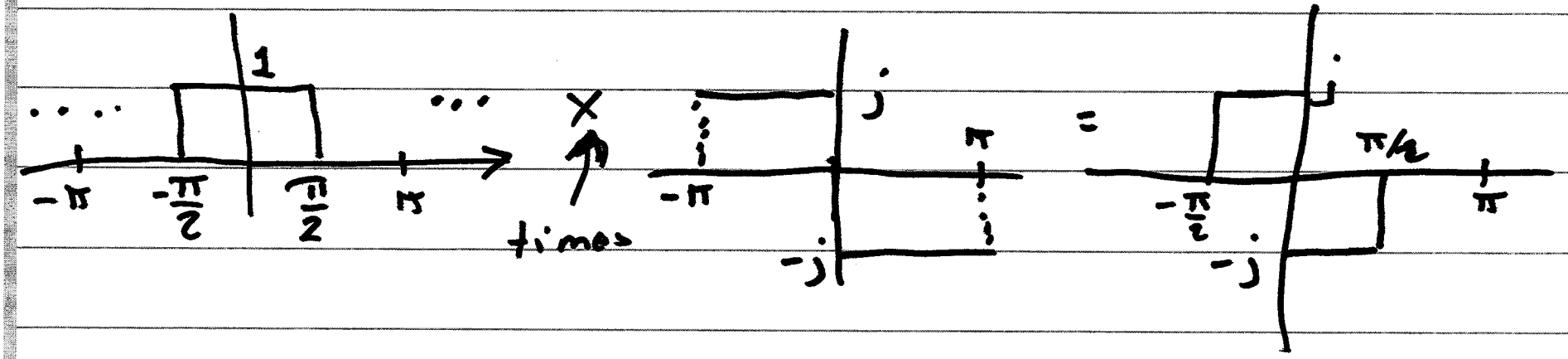
$$\frac{\sin\left(\pi\left(n+\frac{3}{4}\right)\right)}{\pi\left(n+\frac{3}{4}\right)} * \frac{\sin\left(\pi\left(n-\frac{1}{2}\right)\right)}{\pi\left(n-\frac{1}{2}\right)} = ??$$

for $-\pi < \omega < \pi$: $e^{j\frac{3}{4}\omega} e^{-j\frac{1}{2}\omega} = e^{j\frac{1}{4}\omega}$

ANS: $\frac{\sin\left(\pi\left(n+\frac{1}{4}\right)\right)}{\pi\left(n+\frac{1}{4}\right)}$

$h_{HB}[n]$

$$\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} * \overbrace{2\frac{\sin^2\left(\frac{\pi}{2}n\right)}{\pi n}} = ??$$



ANS: $2\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\frac{1}{2}\left\{\frac{1}{j}e^{j\frac{\pi}{4}n} - \frac{1}{j}e^{-j\frac{\pi}{4}n}\right\}$

$$= 2\frac{\sin^2\left(\frac{\pi}{4}n\right)}{\pi n}$$