Continuous-Time Fourier Transform (Hz): $X(F) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$

Continuous-Time Fourier Transform Pair (Hz): $\mathcal{F}\left\{\frac{\sin(2\pi Wt)}{\pi t}\right\} = \text{rect}\left\{\frac{F}{2W}\right\}$ where \(\text{rect}(x) = 1\) for \(|x| < 0.5\) and \(\text{rect}(x) = 0\) for \(|x| > 0.5\).

Continuous-Time Fourier Transform Property: $\mathcal{F}\{x_1(t)x_2(t)\} = X_1(F) * X_2(F)$, where $*$ denotes convolution, and $\mathcal{F}\{x_i(t)\} = X_i(F)$, $i = 1, 2$.

Relationship between DTFT and CTFT frequency variables in Hz: $\omega = 2\pi \frac{F}{F_s}$, where $F_s = \frac{1}{T_s}$ is the sampling rate in Hz.
Problem 1. Consider the upsampler system below in Figure 1.

(a) Draw block diagram of efficient implementation of the upsampler system in Fig. 1.

(b) Your answer to part (a) should involve the polyphase components of $h[n]$: $h_0[n] = h[4n]$, $h_1[n] = h[4n + 1]$, $h_2[n] = h[4n + 2]$, and $h_3[n] = h[4n + 3]$. For the plots requested below, you can do all magnitude plots on one graph and you can do all phase plots on one graph, to save time and space.

(i) For the ideal case where $h[n] = 4\frac{\sin(\frac{\pi}{4}n)}{\pi n}$, plot the magnitude of the DTFT of $h_0[n] = h[4n]$, $H_0(\omega)$, over $-\pi < \omega < \pi$.

(ii) For the general case where $h[n]$ is an arbitrary impulse response, express the DTFT of $h_1[n] = h[4n + 1]$, denoted $H_1(\omega)$, in terms of $H(\omega)$.

(iii) For the ideal case where $h[n] = 4\frac{\sin(\frac{\pi}{4}n)}{\pi n}$, plot both the magnitude AND phase of the DTFT $h_1[n] = h[4n + 1]$, $H_1(\omega)$, over $-\pi < \omega < \pi$.

(iv) Express the DTFT of $h_2[n] = h[4n + 2]$, denoted $H_2(\omega)$, in terms of $H(\omega)$.

(v) For the ideal case where $h[n] = 4\frac{\sin(\frac{\pi}{4}n)}{\pi n}$, plot both the magnitude AND phase of the DTFT $h_2[n] = h[4n + 2]$, $H_2(\omega)$, over $-\pi < \omega < \pi$.

(vi) For the ideal case where $h[n] = 4\frac{\sin(\frac{\pi}{4}n)}{\pi n}$, plot both the magnitude AND phase of the DTFT $h_3[n] = h[4n + 3]$, $H_3(\omega)$, over $-\pi < \omega < \pi$.

(c) Consider that the input to the system in Figure 1 is a sampled version of the analog signal in Figure 2. For the remaining parts of this problem, the input signal is as defined below where $x_a(t)$ is the analog signal in Figure 2. Assume that $1/T_s = 1/2$ is above the Nyquist rate for this signal. That is, even though this signal is not strictly bandlimited, assume that aliasing effects are negligible.

$$x[n] = x_a(nT_s), \quad T_s = 2$$
(i) For the ideal case where \( h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \), determine the output \( y[n] \) of the system in Figure 1, when \( x[n] \) is input to the system. Write output in sequence form (indicating where \( n = 0 \) is) OR do stem plot.

(ii) For the ideal case where \( h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \), determine the output \( y_0[n] = x[n] * h_0[n] \), when \( x[n] \) is input to the filter \( h_0[n] = h[4n] \). Write output in sequence form (indicating where \( n = 0 \) OR do stem plot.

(iii) For the ideal case where \( h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \), determine the output \( y_1[n] = x[n] * h_1[n] \), when \( x[n] \) is input to the filter \( h_1[n] = h[4n + 1] \). Write output in sequence form (indicating where \( n = 0 \) is) OR do stem plot.

(iv) For the ideal case where \( h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \), determine the output \( y_2[n] = x[n] * h_2[n] \), when \( x[n] \) is input to the filter \( h_2[n] = h[4n + 2] \). Write output in sequence form (indicating where \( n = 0 \) is) OR do stem plot.

(v) For the ideal case where \( h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n} \), determine the output \( y_3[n] = x[n] * h_3[n] \), when \( x[n] \) is input to the filter \( h_3[n] = h[4n + 3] \). Write output in sequence form (indicating where \( n = 0 \) is) OR do stem plot.
This page left intentionally blank for student work for Problem 1.
Prob. 2(a) Consider the continuous-time signal $x_0(t)$ below. A discrete-time signal is created by sampling $x_0(t)$ according to $x_0[n] = x_0(nT_s)$ with $F_s = \frac{1}{T_s} = \frac{8}{3}W$. Plot the magnitude of the DTFT of $x_0[n]$, $|X_0(\omega)|$, over $-\pi < \omega < \pi$. Show all work.

$$x_0(t) = T_s \frac{d}{dt} \left\{ \frac{\sin(2\pi W t)}{\pi t} \right\}$$
2(b) Consider the continuous-time signal $x_1(t)$ below. A discrete-time signal is created by sampling $x_1(t)$ according to $x_1[n] = x_1(nT_s)$ with $F_s = \frac{1}{T_s} = \frac{8}{3} W$. Plot the magnitude of the DTFT of $x_1[n]$, $|X_1(\omega)|$, over $-\pi < \omega < \pi$. Show all work.

\[
x_1(t) = T_s \frac{1}{2} \left\{ \frac{\sin(2\pi W t)}{\pi t} + \frac{\sin(2\pi W \frac{t}{3})}{\pi t} \right\}
\]
2(c) Consider the continuous-time signal $x_2(t)$ below. A discrete-time signal is created by sampling $x_2(t)$ according to $x_2[n] = x_2(nT_s)$ with $F_s = \frac{1}{T_s} = 4W$. Plot the magnitude of the DTFT of $x_2[n]$, $|X_2(\omega)|$, over $-\pi < \omega < \pi$. Show all work.

$$x_2(t) = T_s \left\{ \frac{\sin(2\pi \frac{W}{4} t)}{\pi t} \frac{\sin(2\pi \frac{3W}{4} t)}{\pi t} \right\} \cos(2\pi W t)$$
2(d) Consider the continuous-time signal $x_3(t)$ below. A discrete-time signal is created by sampling $x_3(t)$ according to $x_3[n] = x_3(nT_s)$ with $F_s = \frac{1}{T_s} = 4W$. Plot the magnitude of the DTFT of $x_3[n]$, $|X_3(\omega)|$, over $-\pi < \omega < \pi$. Show all work.

$$x_3(t) = T_s \frac{2}{W} \left\{ \frac{\sin \left( \frac{2\pi W t}{\pi t} \right)}{\pi t} \right\}^2 \cos(2\pi W t)$$
NAME: Page intentionally blank for Problem 2(d) Work
Problem 2. This problem is about digital subbanding of the four DT signals $x_i[n]$, $i = 0, 1, 2, 3$ from Problem 2. Digital subbanding of these four signals is effected in an efficient way via filter bank in Figure 3. All of the quantities in Figure 1 are defined below: the respective impulse responses of the polyphase component filters are defined in terms of the ideal lowpass filter impulse response below.

$$h_{LP}[n] = 4 \sin \left( \frac{\pi n}{4} \right)$$  \hspace{1cm} (1)

The polyphase component filters on the left side of Figure 3 are defined as

$$h^+_{\ell}[n] = h_{LP}[4n + \ell], \quad \ell = 0, 1, 2, 3.$$  \hspace{1cm} (2)

The respective signals at the inputs to these filters are formed from the input signals as (from Problem 3) as described below. There is only ONE part to this problem: plot the magnitude of the DTFT $Y(\omega)$ of the interleaved signal $y[n]$.

$$y_0[n] = x_0[n] + x_1[n] \cos \left( \frac{\pi}{2} \right) - \hat{x}_1[n] \sin \left( \frac{\pi}{2} \right) + x_2[n] \cos \left( \frac{\pi}{2} \right) + \hat{x}_2[n] \sin \left( \frac{\pi}{2} \right) + x_3[n] \cos(\pi)$$  \hspace{1cm} (3)

$$y_1[n] = x_0[n] + x_1[n] \cos \left( \frac{\pi}{2} \right) - \hat{x}_1[n] \sin \left( \frac{\pi}{2} \right) + x_2[n] \cos \left( \frac{\pi}{2} \right) + \hat{x}_2[n] \sin \left( \frac{\pi}{2} \right) + x_3[n] \cos(\pi)$$  \hspace{1cm} (3)

$$y_2[n] = x_0[n] + x_1[n] \cos \left( \frac{\pi}{2} \right) - \hat{x}_1[n] \sin \left( \frac{\pi}{2} \right) + x_2[n] \cos \left( \frac{\pi}{2} \right) + \hat{x}_2[n] \sin \left( \frac{\pi}{2} \right) + x_3[n] \cos(\pi)$$  \hspace{1cm} (3)

$$y_3[n] = x_0[n] + x_1[n] \cos \left( \frac{\pi}{2} \right) - \hat{x}_1[n] \sin \left( \frac{\pi}{2} \right) + x_2[n] \cos \left( \frac{\pi}{2} \right) + \hat{x}_2[n] \sin \left( \frac{\pi}{2} \right) + x_3[n] \cos(\pi)$$  \hspace{1cm} (3)