

**NAME:** **28 Oct. 2016**  
**ECE 538 Digital Signal Processing I Exam 2 Fall 2016**

## Cover Sheet

**WRITE YOUR NAME ON THIS COVER SHEET**

Test Duration: 60 minutes.

Open Book but Closed Notes.

One (both sides) handwritten 8.5 in x 11 in crib sheet allowed

Calculators NOT allowed.

All work should be done in the space provided.

There are three problems.

**Continuous-Time Fourier Transform (Hz):**  $X(F) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft} dt$

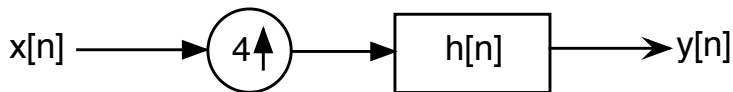
**Continuous-Time Fourier Transform Pair (Hz):**  $\mathcal{F}\left\{\frac{\sin(2\pi Wt)}{\pi t}\right\} = \text{rect}\left\{\frac{F}{2W}\right\}$  where

$\text{rect}(x) = 1$  for  $|x| < 0.5$  and  $\text{rect}(x) = 0$  for  $|x| > 0.5$ .

**Continuous-Time Fourier Transform Property:**  $\mathcal{F}\{x_1(t)x_2(t)\} = X_1(F) * X_2(F)$ , where  $*$  denotes convolution, and  $\mathcal{F}\{x_i(t)\} = X_i(F)$ ,  $i = 1, 2$ .

**Relationship between DTFT and CTFT frequency variables in Hz:**  $\omega = 2\pi\frac{F}{F_s}$ , where  $F_s = \frac{1}{T_s}$  is the sampling rate in Hz

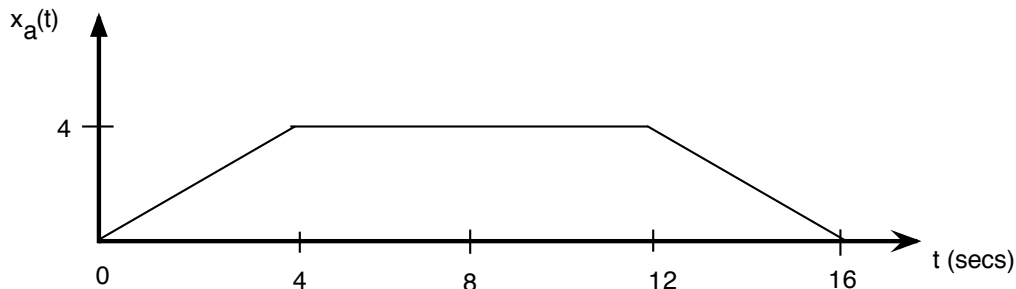
**Problem 1.** Consider the upsampler system below in Figure 1.



**Figure 1.**

- (a) Draw block diagram of efficient implementation of the upsampler system in Fig. 1.
- (b) Your answer to part (a) should involve the polyphase components of  $h[n]$ :  $h_0[n] = h[4n]$ ,  $h_1[n] = h[4n + 1]$ ,  $h_2[n] = h[4n + 2]$ , and  $h_3[n] = h[4n + 3]$ . For the plots requested below, you can do all magnitude plots on one graph and you can do all phase plots on one graph, to save time and space.
- (i) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , plot the magnitude of the DTFT of  $h_0[n] = h[4n]$ ,  $H_0(\omega)$ , over  $-\pi < \omega < \pi$ .
  - (ii) For the general case where  $h[n]$  is an arbitrary impulse response, express the DTFT of  $h_1[n] = h[4n + 1]$ , denoted  $H_1(\omega)$ , in terms of  $H(\omega)$ .
  - (iii) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , plot both the magnitude AND phase of the DTFT  $h_1[n] = h[4n + 1]$ ,  $H_1(\omega)$ , over  $-\pi < \omega < \pi$ .
  - (iv) Express the DTFT of  $h_2[n] = h[4n + 2]$ , denoted  $H_2(\omega)$ , in terms of  $H(\omega)$ .
  - (v) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , plot both the magnitude AND phase of the DTFT  $h_2[n] = h[4n + 2]$ ,  $H_2(\omega)$ , over  $-\pi < \omega < \pi$ .
  - (vi) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , plot both the magnitude AND phase of the DTFT  $h_3[n] = h[4n + 3]$ ,  $H_3(\omega)$ , over  $-\pi < \omega < \pi$ .
- (c) Consider that the input to the system in Figure 1 is a sampled version of the analog signal in Figure 2. For the remaining parts of this problem, the input signal is as defined below where  $x_a(t)$  is the analog signal in Figure 2. Assume that  $1/T_s = 1/2$  is above the Nyquist rate for this signal. That is, even though this signal is not strictly bandlimited, assume that aliasing effects are negligible.

$$x[n] = x_a(nT_s), \quad T_s = 2$$



**Figure 2.**

- (i) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , determine the output  $y[n]$  of the system in Figure 1, when  $x[n]$  is input to the system. Write output in sequence form (indicating where  $n = 0$  is) OR do stem plot.
- (ii) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , determine the output  $y_0[n] = x[n] * h_0[n]$ , when  $x[n]$  is input to the filter  $h_0[n] = h[4n]$ . Write output in sequence form (indicating where is  $n = 0$  OR do stem plot.
- (iii) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , determine the output  $y_1[n] = x[n] * h_1[n]$ , when  $x[n]$  is input to the filter  $h_1[n] = h[4n + 1]$ . Write output in sequence form (indicating where  $n = 0$  is) OR do stem plot.
- (iv) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , determine the output  $y_2[n] = x[n] * h_2[n]$ , when  $x[n]$  is input to the filter  $h_2[n] = h[4n + 2]$ . Write output in sequence form (indicating where  $n = 0$  is) OR do stem plot.
- (v) For the ideal case where  $h[n] = 4 \frac{\sin(\frac{\pi}{4}n)}{\pi n}$ , determine the output  $y_3[n] = x[n] * h_3[n]$ , when  $x[n]$  is input to the filter  $h_3[n] = h[4n + 3]$ . Write output in sequence form (indicating where  $n = 0$  is) OR do stem plot.

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Prob. 2(a) Consider the continuous-time signal  $x_0(t)$  below. A discrete-time signal is created by sampling  $x_0(t)$  according to  $x_0[n] = x_0(nT_s)$  with  $F_s = \frac{1}{T_s} = \frac{8}{3}W$ . Plot the magnitude of the DTFT of  $x_0[n]$ ,  $|X_0(\omega)|$ , over  $-\pi < \omega < \pi$ . Show all work.

$$x_0(t) = T_s \frac{d}{dt} \left\{ \frac{\sin(2\pi Wt)}{\pi t} \right\}$$

2(b) Consider the continuous-time signal  $x_1(t)$  below. A discrete-time signal is created by sampling  $x_1(t)$  according to  $x_1[n] = x_1(nT_s)$  with  $F_s = \frac{1}{T_s} = \frac{8}{3}W$ . Plot the magnitude of the DTFT of  $x_1[n]$ ,  $|X_1(\omega)|$ , over  $-\pi < \omega < \pi$ . Show all work.

$$x_1(t) = T_s \frac{1}{2} \left\{ \frac{\sin(2\pi Wt)}{\pi t} + \frac{\sin(2\pi \frac{W}{3}t)}{\pi t} \right\}$$



2(c) Consider the continuous-time signal  $x_2(t)$  below. A discrete-time signal is created by sampling  $x_2(t)$  according to  $x_2[n] = x_2(nT_s)$  with  $F_s = \frac{1}{T_s} = 4W$ . Plot the magnitude of the DTFT of  $x_2[n]$ ,  $|X_2(\omega)|$ , over  $-\pi < \omega < \pi$ . Show all work.

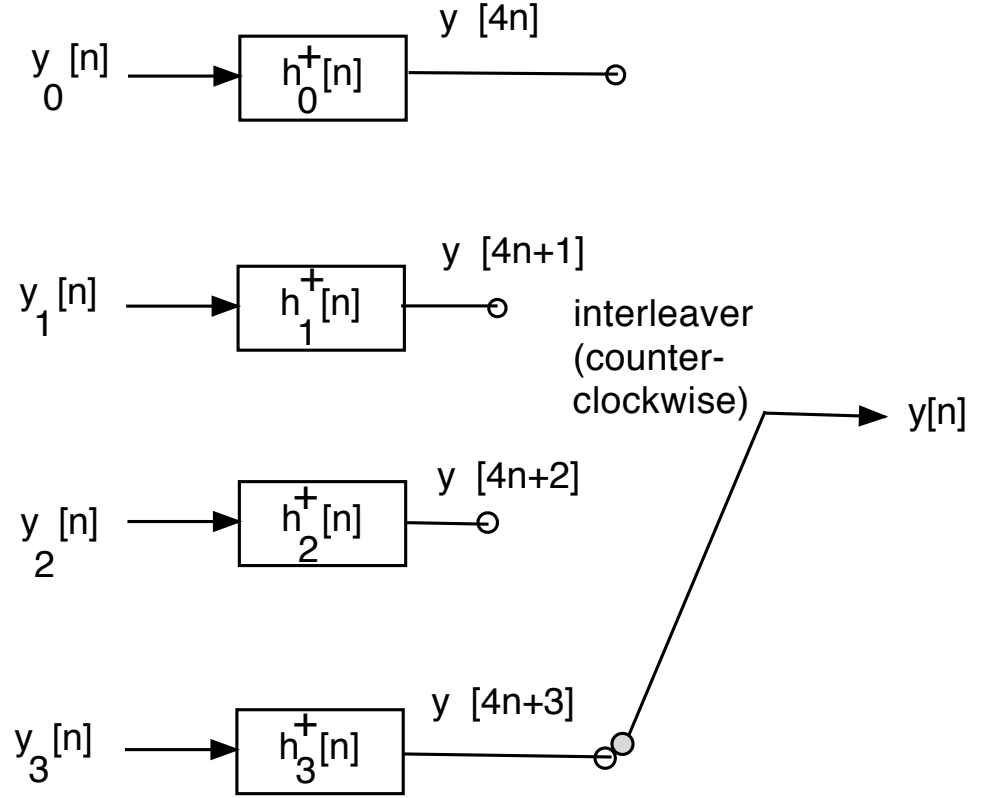
$$x_2(t) = T_s \left\{ \frac{\sin(2\pi \frac{W}{4} t)}{\pi t} \frac{\sin(2\pi \frac{3W}{4} t)}{\pi t} \right\} \cos(2\pi W t)$$

- 2(d) Consider the continuous-time signal  $x_3(t)$  below. A discrete-time signal is created by sampling  $x_3(t)$  according to  $x_3[n] = x_3(nT_s)$  with  $F_s = \frac{1}{T_s} = 4W$ . Plot the magnitude of the DTFT of  $x_3[n]$ ,  $|X_3(\omega)|$ , over  $-\pi < \omega < \pi$ . Show all work.

$$x_3(t) = T_s \frac{2}{W} \left\{ \frac{\sin\left(2\pi \frac{W}{2} t\right)}{\pi t} \right\}^2 \cos(2\pi W t)$$

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**Figure 3.**

**Problem 2.** This problem is about digital subbanding of the four DT signals  $x_i[n]$ ,  $i = 0, 1, 2, 3$  from Problem 2. Digital subbanding of these four signals is effected in an efficient way via filter bank in Figure 3. All of the quantities in Figure 1 are defined below: the respective impulse responses of the polyphase component filters are defined in terms of the ideal lowpass filter impulse response below.

$$h_{LP}[n] = 4 \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \quad (1)$$

The polyphase component filters on the left side of Figure 3 are defined as

$$h_\ell^+[n] = h_{LP}[4n + \ell], \quad \ell = 0, 1, 2, 3. \quad (2)$$

The respective signals at the inputs to these filters are formed from the input signals as (from Problem 3) as described below. **There is only ONE part to this problem: plot the magnitude of the DTFT  $Y(\omega)$  of the interleaved signal  $y[n]$ .**

$$\begin{aligned} y_0[n] &= x_0[n] + x_1[n] \cos\left(\frac{\pi}{2}0\right) - \hat{x}_1[n] \sin\left(\frac{\pi}{2}0\right) + x_2[n] \cos\left(\frac{\pi}{2}0\right) + \hat{x}_2[n] \sin\left(\frac{\pi}{2}0\right) + x_3[n] \cos(\pi 0) \\ y_1[n] &= x_0[n] + x_1[n] \cos\left(\frac{\pi}{2}1\right) - \hat{x}_1[n] \sin\left(\frac{\pi}{2}1\right) + x_2[n] \cos\left(\frac{\pi}{2}1\right) + \hat{x}_2[n] \sin\left(\frac{\pi}{2}1\right) + x_3[n] \cos(\pi 1) \\ y_2[n] &= x_0[n] + x_1[n] \cos\left(\frac{\pi}{2}2\right) - \hat{x}_1[n] \sin\left(\frac{\pi}{2}2\right) + x_2[n] \cos\left(\frac{\pi}{2}2\right) + \hat{x}_2[n] \sin\left(\frac{\pi}{2}2\right) + x_3[n] \cos(\pi 2) \\ y_3[n] &= x_0[n] + x_1[n] \cos\left(\frac{\pi}{2}3\right) - \hat{x}_1[n] \sin\left(\frac{\pi}{2}3\right) + x_2[n] \cos\left(\frac{\pi}{2}3\right) + \hat{x}_2[n] \sin\left(\frac{\pi}{2}3\right) + x_3[n] \cos(\pi 3) \end{aligned} \quad (3)$$

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