NAME: **31 Oct. 2011**
ECE 538 Digital Signal Processing I Exam 2 Fall 2011

Cover Sheet

WRITE YOUR NAME ON THIS COVER SHEET
Test Duration: 60 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
All work should be done in the space provided.

Problem 1. Consider four different signals with the spectral shapes shown in Figure 1 on the next page, each of which is sampled at the Nyquist rate. Digital subbanding of the the DT signals is effected via the system in the block diagram in Figure 1. The respective impulse responses for the four filters

$$f_k[n] = e^{j(k\frac{2\pi}{4})n} h_{LP}[n], \quad k = 0, 1, 2, 3$$

are defined in terms of the ideal lowpass filter below

$$h_{LP}[n] = 4\frac{\sin(\frac{\pi}{4}n)}{\pi n}. \tag{1}$$

(a) Plot the magnitude of the DTFT $Y(\omega)$ of the final sum output $y[n]$ over $-\pi < \omega < \pi$. You will lose credit if you just do the plot with no supporting work and/or explanation. At the same time, though, be concise and don't overdo it as you have to save time for the other parts of this exam. Note that $\frac{6\pi}{4} = \frac{3\pi}{2}$ is the same DT frequency as $-\frac{2\pi}{4} = -\frac{\pi}{2}$.

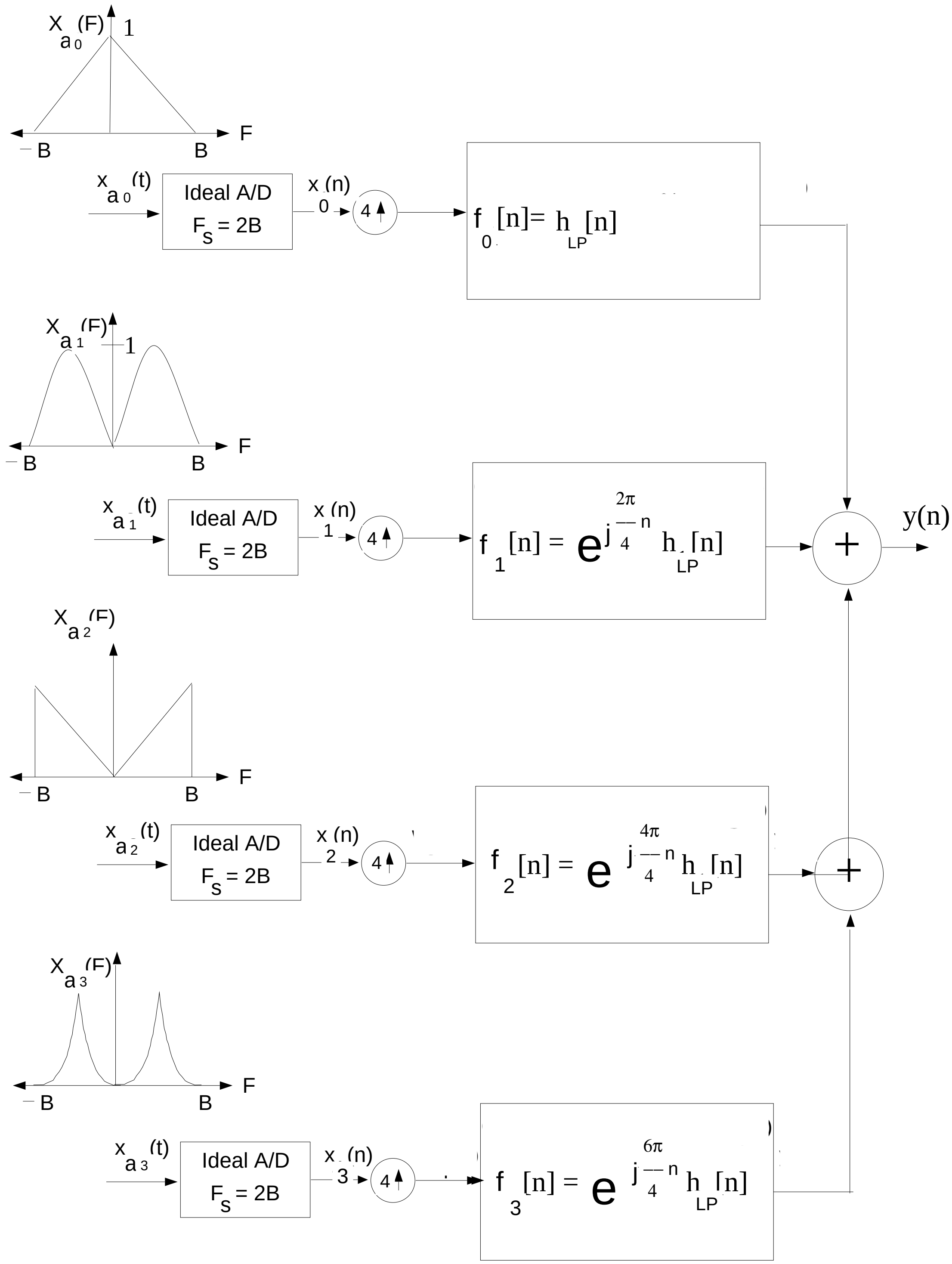

Figure 1 Digital subbanding of four real-valued signals each sampled at Nyquist rate.

(b) The four polyphase components of $h_{LP}[n]$ are defined as $h_\ell[n] = h_{LP}[4n+\ell]$, $\ell = 0, 1, 2, 3$, such that

$$h_\ell[n] = 4\frac{\sin\left(\frac{\pi}{4}(4n+\ell)\right)}{\pi(4n+\ell)} = \frac{\sin\left(\pi(n+\frac{\ell}{4})\right)}{\pi(n+\frac{\ell}{4})}, \quad \ell = 0, 1, 2, 3 \tag{2}$$

The frequency response of each filter is $H_\ell(\omega)$ (the DTFT of $h_\ell[n]$) $\ell = 0, 1, 2, 3$. Plot the phase of each filter $\angle H_\ell(\omega)$ over $-\pi < \omega < \pi$, $\ell = 0, 1, 2, 3$. You can either do them all on one plot or on separate plots. Clearly indicate the slope in each case.

(c) Digital subbanding is alternatively effected more efficiently with the polyphase filters via the block diagram in Figure 2 where the respective input signals are defined as below. Determine the value of each of the amplitude coefficients $\alpha_{\ell k}$, $k = 1, 2, 3$, $\ell = 1, 2, 3$ so that the output $y[n]$ is the SAME as the output in Figure 1. Show all work on the next page but input your final answers in the table below. You will lose credit for answers that are not justified by work and/or explanation.

$$
\begin{aligned}
s_0[n] &= x_0[n] + x_1[n] + x_2[n] + x_3[n] \\
s_1[n] &= x_0[n] + \alpha_{11}x_1[n] + \alpha_{12}x_2[n] + \alpha_{13}x_3[n] \\
s_2[n] &= x_0[n] + \alpha_{21}x_1[n] + \alpha_{22}x_2[n] + \alpha_{23}x_3[n] \\
s_3[n] &= x_0[n] + \alpha_{31}x_1[n] + \alpha_{32}x_2[n] + \alpha_{33}x_3[n]
\end{aligned}
$$

$\alpha_{\ell k}$	$k=1$	$k=2$	$k=3$
$\ell = 1$			
$\ell = 2$			
$\ell = 3$			

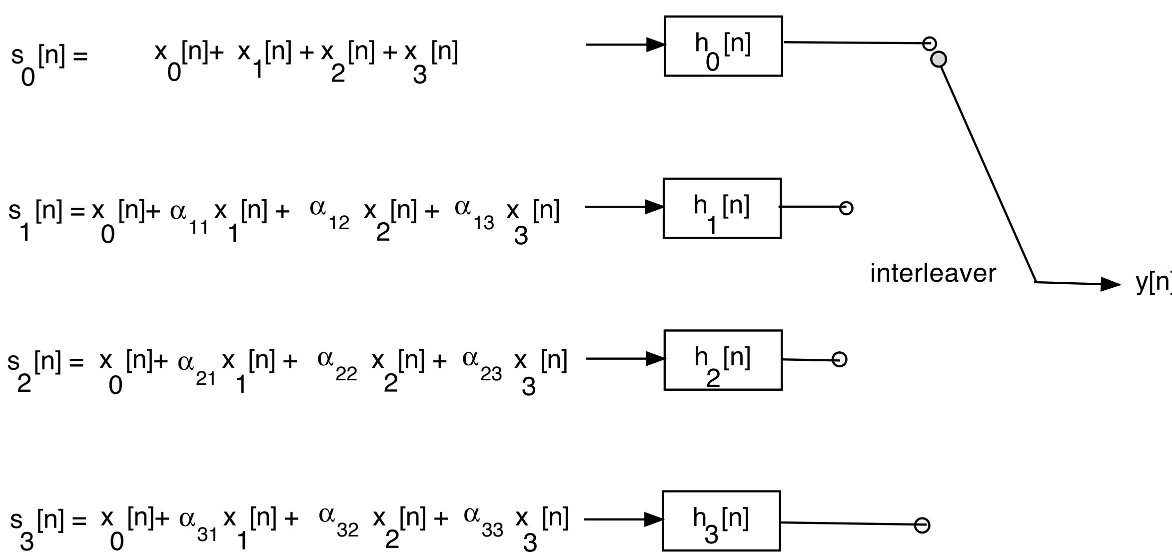

Figure 2.

(d) The system depicted in Figure 3 is employed to recover the four original signals back from the interleaved sum $y[n]$ in Figure 2. Fill in the values of $\beta_{k\ell}$ in the table below so that $z_k[n] = x_k[n], k = 0, 1, 2, 3$. Show all work on the next page.

$$
\begin{aligned}
z_0[n] &= g_0[n] + g_1[n] + g_2[n] + g_3[n] \\
z_1[n] &= g_0[n] + \beta_{11} g_1[n] + \beta_{12} g_2[n] + \beta_{13} g_3[n] \\
z_2[n] &= g_0[n] + \beta_{21} g_1[n] + \beta_{22} g_2[n] + \beta_{23} g_3[n] \\
z_3[n] &= g_0[n] + \beta_{31} g_1[n] + \beta_{32} g_2[n] + \beta_{33} g_3[n]
\end{aligned}
$$

$\beta_{k\ell}$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$k = 1$			
$k = 2$			
$k = 3$			

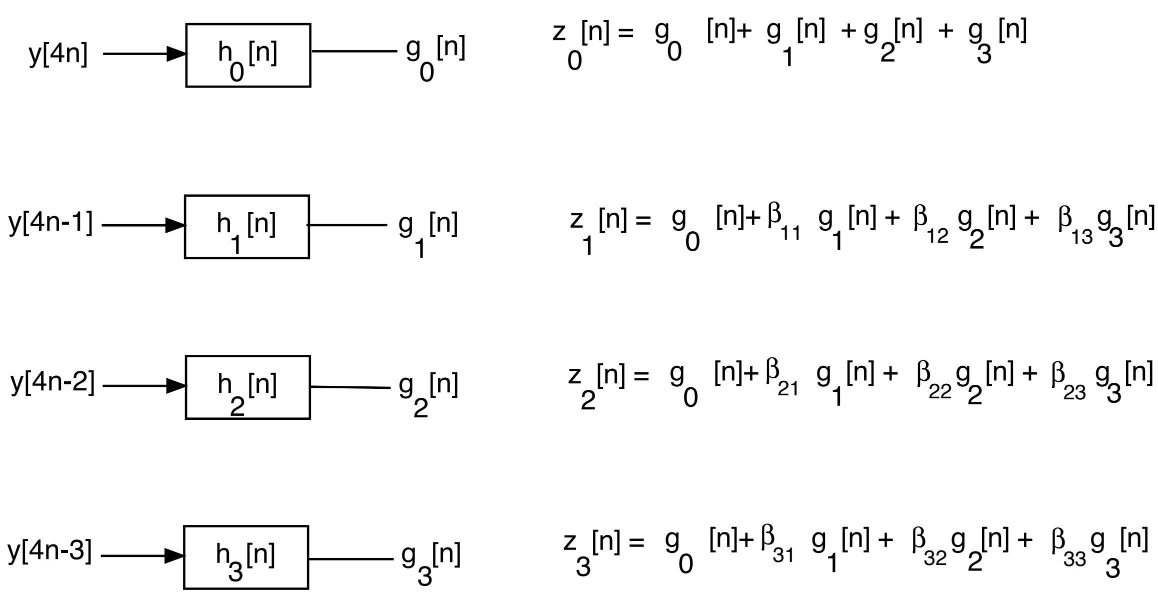

Figure 3.

(e) Consider 8 different real-valued signals, $v_k[n]$, $k = 1, 2, ..., 8$, that were sampled near the Nyquist rate. Let $\hat{v}_k[n]$ denote the respective Hilbert Transform of each signal. We form the following four complex-valued signals:

$$x_0[n] = (v_1[n] + j\hat{v}_1[n]) + (v_2[n] - j\hat{v}_2[n])$$

$$x_1[n] = (v_3[n] + j\hat{v}_3[n]) + (v_4[n] - j\hat{v}_4[n])$$

$$x_2[n] = (v_5[n] + j\hat{v}_5[n]) + (v_6[n] - j\hat{v}_6[n])$$

$$x_3[n] = (v_7[n] + j\hat{v}_7[n]) + (v_8[n] - j\hat{v}_8[n])$$

These four signals are the respective inputs to the system in Figure 1 to form the sum signal $y[n]$. For each of the 8 signals, $v_k[n]$, $k = 1, 2, ..., 8$, indicate the frequency band that it occupies in the sum signal $y[n]$ within $-\pi < \omega < \pi$. I filled in the first one for you. You have to put in the correct answer for the remaining seven signals.

Signal	Located in Frequency Band:
$v_1[n]$	$0 < \omega < \frac{\pi}{4}$
$v_2[n]$	
$v_3[n]$	
$v_4[n]$	
$v_5[n]$	
$v_6[n]$	
$v_7[n]$	
$v_8[n]$	