EE538 Digital Signal Processing I **Session 27**
Exam 2 **Live: Friday, Oct. 29, 2010**

Cover Sheet

Test Duration: 55 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems and four pages including this cover page.
All work should be done on loose blank sheets of 8 1/2 x 11 paper
which will NOT be provided. Write on only one side of each sheet
and then staple together.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the exam sheets.

No.	**Topic(s) of Problem**	**Points**
1.	Perfect Reconstruction Filter Bank - Two Channel	45
2.	Vestigial Sideband Modulation	55

Digital Signal Processing I Exam 2 Fall 2010

Problem 1. [45 points]

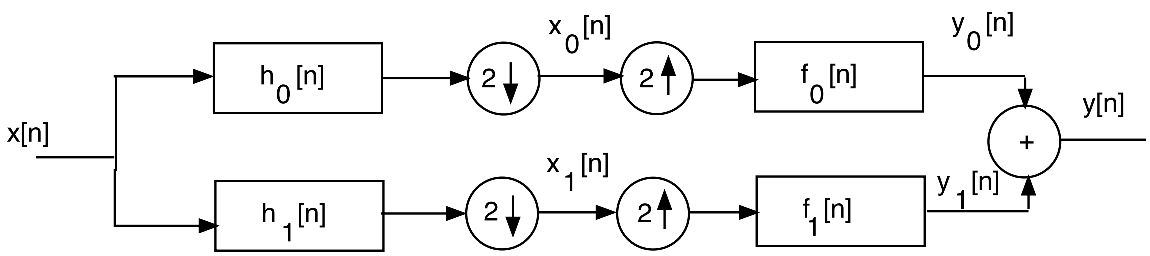
In the system below, the two analysis filters, $h_0[n]$ and $h_1[n]$, and the two synthesis filters, $f_0[n]$ and $f_1[n]$, form a Quadrature Mirror Filter (QMF). Specifically,

$$h_0[n] = \frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}, -\infty < n < \infty \text{ with } \beta = 0.5 \tag{1}$$

$$h_1[n] = (-1)^n h_0[n] \qquad f_0[n] = h_0[n] \qquad f_1[n] = -h_1[n]$$

The DTFT of the halfband filter $h_0[n]$ above may be expressed as follows:

$$H_0(\omega) = \begin{cases} e^{j\frac{\omega}{2}}, & |\omega| < \frac{\pi}{4} \\ e^{j\frac{\omega}{2}}\cos[(|\omega| - \frac{\pi}{4})], & \frac{\pi}{4} < |\omega| < \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$



Consider the following input signal

$$x[n] = 16\frac{\sin\left(\frac{3\pi}{8}n\right)}{\pi n}\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\cos\left(\frac{\pi}{2}n\right)$$

(a) Plot the magnitude of the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$.

(b) Plot the magnitude of the DTFT of $x_0[n]$, $X_0(\omega)$, over $-\pi < \omega < \pi$.

(c) Plot the magnitude of the DTFT of $x_1[n]$, $X_1(\omega)$, over $-\pi < \omega < \pi$.

(d) Plot the magnitude of the DTFT of $y_0[n]$, $Y_0(\omega)$, over $-\pi < \omega < \pi$.

(e) Plot the magnitude of the DTFT of $y_1[n]$, $Y_1(\omega)$, over $-\pi < \omega < \pi$.

(f) Plot the magnitude of the DTFT of the final output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

Problem 2. [55 points] This problem is about Vestigial Sideband (VSB) Modulation. Consider the following complex-valued filter.

$$h[n] = e^{j\frac{\pi}{3}n} h_{LP}[n]$$

Let $H_{LP}(\omega)$ denote the DTFT (frequency response) of the filter $h_{LP}[n]$. $H_{LP}(\omega)$ is real-valued and symmetric. For $-\pi < \omega < \pi$, $H_{LP}(\omega)$ is mathematically described as

$$H_{LP}(\omega) = \begin{cases} 2, & |\omega| < \frac{\pi}{4} \\ 1 + \cos[6(|\omega| - \frac{\pi}{4})], & \frac{3\pi}{12} < |\omega| < \frac{5\pi}{12} \\ 0, & \frac{5\pi}{12} < |\omega| < \pi \end{cases}$$

Note that for later purposes in this problem, we can break up $h[n]$ into its real and imaginary parts using Euler's formula as $h[n] = h_R[n] + jh_I[n]$, where:

$$h_R[n] = Re\{h[n]\} = h_{LP}[n] \cos\left(\frac{\pi}{3}n\right) \qquad h_I[n] = Im\{h[n]\} = h_{LP}[n] \sin\left(\frac{\pi}{3}n\right)$$

(a) Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) Consider the following input signal

$$x[n] = 4\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\frac{\sin\left(\frac{\pi}{6}n\right)}{\pi n}$$

Plot the magnitude of the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$.

(c) The signal in part (b) is run through the filter in part (a) to produce the output $y[n]$

$$y[n] = x[n] * h[n]$$

Plot the magnitude of the DTFT of $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

(d) Consider the real part of the complex-valued filter $h[n]$. Using Euler's formula, we have that

$$h_R[n] = Re\{h[n]\} = h_{LP}[n] \cos\left(\frac{\pi}{3}n\right)$$

Plot the magnitude of the DTFT of $h_R[n]$, $H_R(\omega)$, over $-\pi < \omega < \pi$.

(e) Consider the imaginary part of the complex-valued filter $h[n]$. Using Euler's formula, we have that

$$h_I[n] = Im\{h[n]\} = h_{LP}[n] \sin\left(\frac{\pi}{3}n\right)$$

Plot the magnitude of the DTFT of $h_I[n]$, $H_I(\omega)$, over $-\pi < \omega < \pi$. **VIP:** Remember that the magnitude of a sum is NOT equal to the sum of the magnitudes, so be careful.

(f) Consider the real part of the complex-valued signal $y[n]$. Since the input $x[n]$ is real-valued, from part (d) we have:

$$y_R[n] = Re\{y[n]\} = x[n] * h_R[n]$$

Plot the magnitude of the DTFT of $y_R[n]$ over $-\pi < \omega < \pi$.

(g) Consider the imaginary part of the complex-valued signal $y[n]$. Since the input $x[n]$ is real-valued, from part (e) we have:

$$y_I[n] = Im\{y[n]\} = x[n] * h_I[n]$$

Plot the magnitude of the DTFT of $y_I[n]$ over $-\pi < \omega < \pi$.

(h) Consider the real-valued signal $z[n]$ formed as

$$z[n] = y_R[n] \cos\left(\frac{\pi}{2}n\right) - y_I[n] \sin\left(\frac{\pi}{2}n\right)$$

Note that $z[n]$ is the real part of $y[n]e^{j\frac{\pi}{2}n}$. Plot the DTFT of $z[n]$ over $-\pi < \omega < \pi$.

(i) Draw a block diagram for a system for recovering the original signal $x[n]$ from $z[n]$.