# ECE 538 Digital Signal Processing I Exam 2 Fall 2009
# 16 Nov. 2009

## Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **TWO** problems.
All work should be done on loose blank 8.5" x 11" sheets (not provided.)
You do **not** have to return this test sheet.

| **Prob. No.** | **Topic of Problem** | **Points** |
|---|---|---|
| 1. | Efficient Upsampling | 60 |
| 2. | Perfect Reconstruction Filter Bank | 40 |

**Problem 1.** [60 points] Consider the upsampler system below in Figure 1.

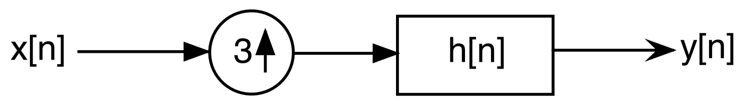


**Figure 1.**

(a) Draw a block diagram of an efficient implementation of the upsampler system in Figure 1.

(b) Your answer to part (a) should involve the polyphase components of $h[n]$: $h_0[n] = h[3n]$, $h_1[n] = h[3n+1]$, and $h_2[n] = h[3n+2]$.

(i) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot the magnitude of the DTFT of $h_0[n] = h[3n]$, $H_0(\omega)$, over $-\pi < \omega < \pi$.

(ii) For the general case where $h[n]$ is an arbitrary impulse response, derive the DTFT of $h_1[n] = h[3n+1]$. Express $H_1(\omega)$ in terms of the DTFT of $h[n]$, $H(\omega)$. Show all steps and all work.

(iii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot both the magnitude AND phase (two separate plots) of the DTFT $h_1[n] = h[3n+1]$, $H_1(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible.

(iv) For the general case where $h[n]$ is an arbitrary impulse response, derive the DTFT of $h_2[n] = h[3n+2]$. Express $H_2(\omega)$ in terms of $H(\omega)$, the DTFT of $h[n]$. Show all steps and all work.

(v) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, plot both the magnitude AND phase (two separate plots) of the DTFT $h_2[n] = h[3n+2]$, $H_2(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible.

(c) Consider that the input to the system in Figure 1 is a sampled version of the analog signal in Figure 2. For the remaining parts of this problem, the input signal is as defined below where $x_a(t)$ is the analog signal in Figure 2. Assume that $1/T_s = 1/3$ is above the Nyquist rate for this signal. That is, even though this signal is not strictly bandlimited, assume that aliasing effects are negligible.

$$x[n] = x_a(nT_s), \qquad T_s = 3$$

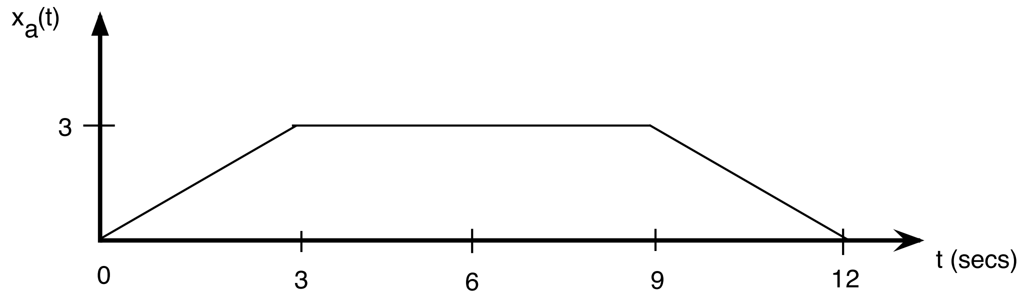


**Figure 2.**

(i) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine and plot the output $y[n]$ of the system in Figure 1, when $x[n]$ is input to the system. Label your plot very carefully.

(ii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine and plot the output $y_0[n] = x[n] * h_0[n]$, when $x[n]$ is input to the filter $h_0[n] = h[3n]$. Label your plot very carefully.

(iii) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine and plot the output $y_1[n] = x[n] * h_1[n]$, when $x[n]$ is input to the filter $h_1[n] = h[3n+1]$. Label your plot very carefully.

(iv) For the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$, determine and plot the output $y_2[n] = x[n] * h_2[n]$, when $x[n]$ is input to the filter $h_2[n] = h[3n+2]$. Label your plot very carefully.

**Problem 2 is on the next page.**

**Problem 2.** [40 points]

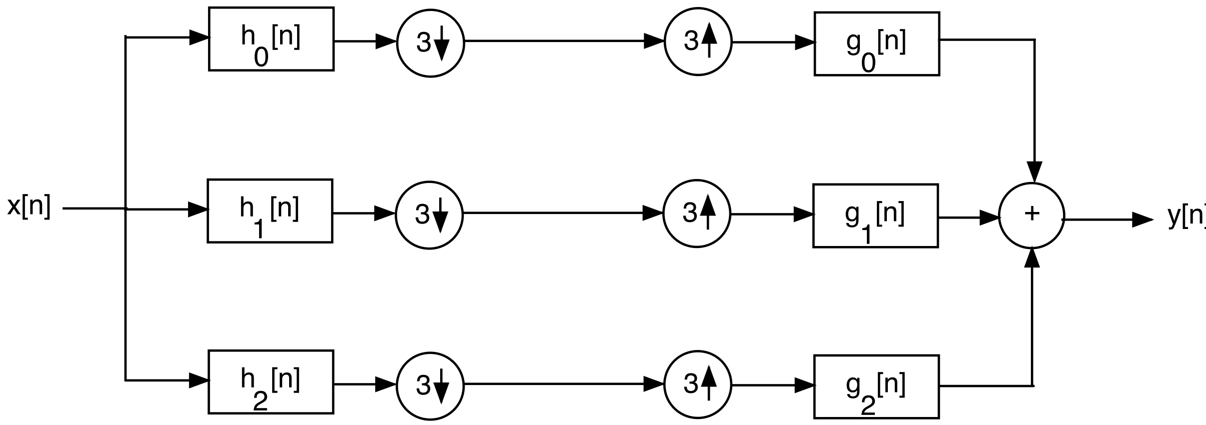


**Figure 3.**

In the frequency domain, the DTFT of the output is related to the DTFT at the input according to the expression below.

$$Y(\omega) = A(\omega)X(\omega) + B(\omega)X\left(\omega - \frac{2\pi}{3}\right) + C(\omega)X\left(\omega - \frac{4\pi}{3}\right)$$

(a) Determine expressions for $A(\omega)$, $B(\omega)$, and $C(\omega)$ in terms of $H_0(\omega)$, $H_1(\omega)$, and $H_2(\omega)$ and $G_0(\omega)$, $G_1(\omega)$, and $G_2(\omega)$.

(b) Consider the case where $g_i[n] = h_i^*[-n]$, $i = 0, 1, 2$. Determine expressions for $A(\omega)$, $B(\omega)$, and $C(\omega)$ in terms of $H_0(\omega)$, $H_1(\omega)$, and $H_2(\omega)$.

(c) Consider the case where $g_i[n] = h_i^*[-n]$, $i = 0, 1, 2$ AND $h_i[n] = e^{-j(i-1)\frac{2\pi}{3}n}h[n]$, $i = 0, 1, 2$. Determine expressions for $A(\omega)$, $B(\omega)$, and $C(\omega)$ all in terms of $H(\omega)$.

(d) **NOTE: this last part of the problem was canceled prior to giving the exam as it was too much work to do in the time allotted.** Consider the case where $g_i[n] = h_i^*[-n]$, $i = 0, 1, 2$ AND $h_i[n] = e^{-j(i-1)\frac{2\pi}{3}n}h[n]$, $i = 0, 1, 2$ AND $h[n] = u[n] - u[n-3]$. Show that $A(\omega) = 9$, $B(\omega) = 0$, and $C(\omega) = 0$ for all $\omega$.