EE538

Exam 2 Soln. Fall 2008

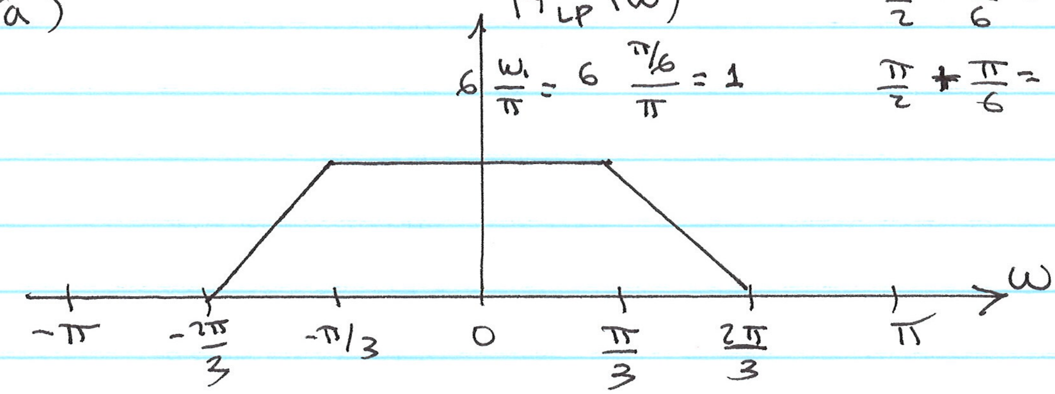
Prob. 1

(a) $H_{LP}(\omega)$

$$6\,\frac{\omega_1}{\pi} = 6\,\frac{\pi/6}{\pi} = 1$$

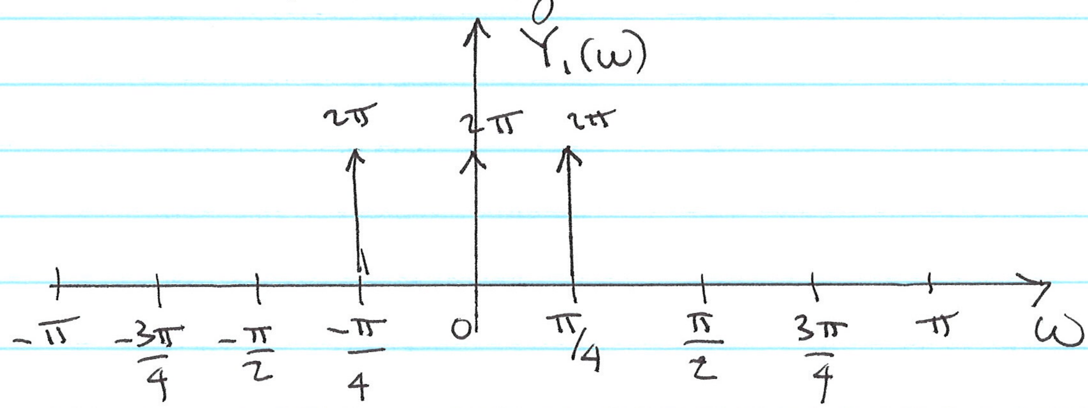
$$\frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi}{6} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$



(b) $\left.Y_1(\omega) \Rightarrow X_1(2\omega)\right\}$ with image centered at $\omega = \pi$ removed

compression by a factor of 2



(c) (i) $h_{LP}^{(0)}[n] = h_{LP}[2n]$

$$= 6\,\frac{\sin\left(\frac{\pi}{2}2n\right)}{\pi 2n}\,\frac{\sin\left(\frac{\pi}{6}2n\right)}{\pi 2n}$$

$$= \frac{6}{4}\,\frac{\sin(\pi n)}{\pi n}\,\underbrace{\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}}_{n=0}$$

$$= \frac{3}{2}\,\delta[n]\left(\frac{1}{3}\right)$$

$$= \frac{1}{2}\,\delta[n]$$

(c)-(ii) $y_1^{(0)}[n] = \frac{1}{2}\delta[n] * x_1[n]$

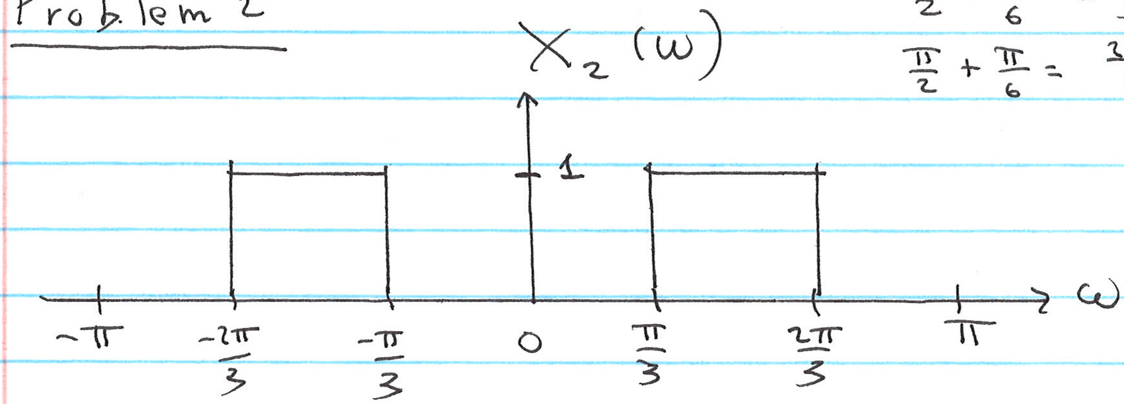
$$= \frac{1}{2} x_1[n]$$

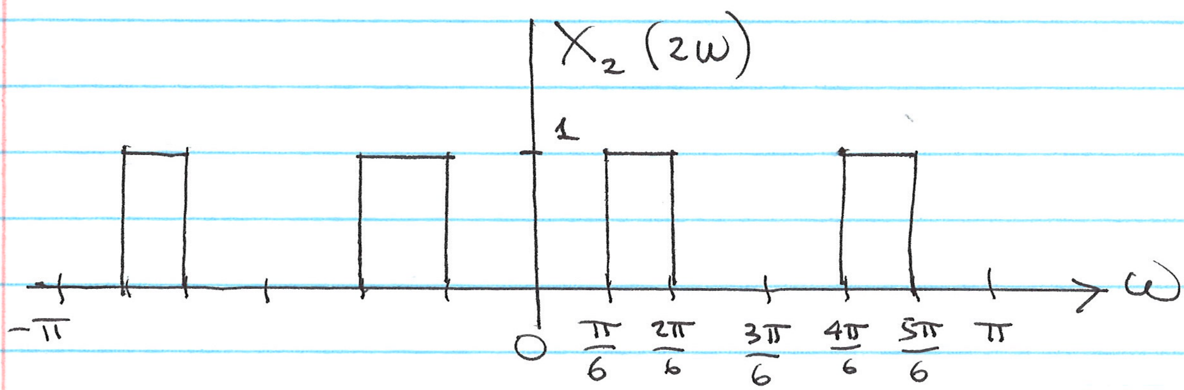
If I had made the gain of $H_{LP}(\omega)$ be 2 over the passband, then $y_1^{(0)}[n] = x_1[n]$

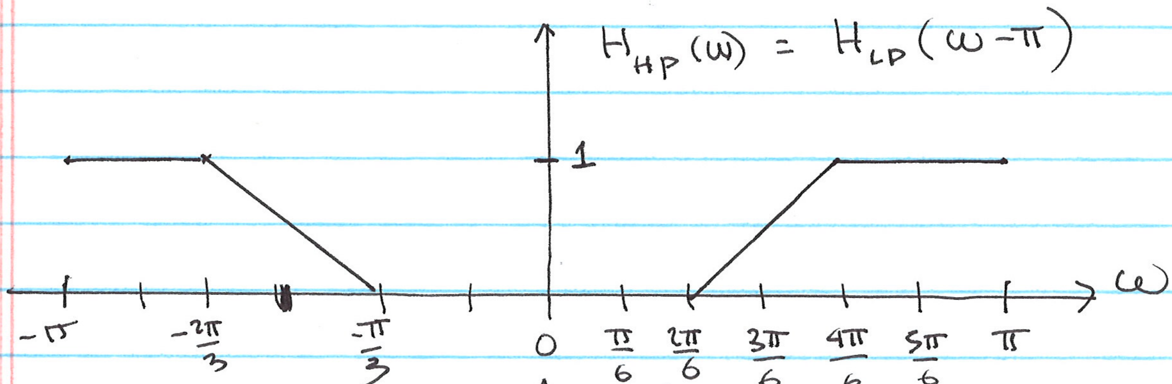
Problem 2

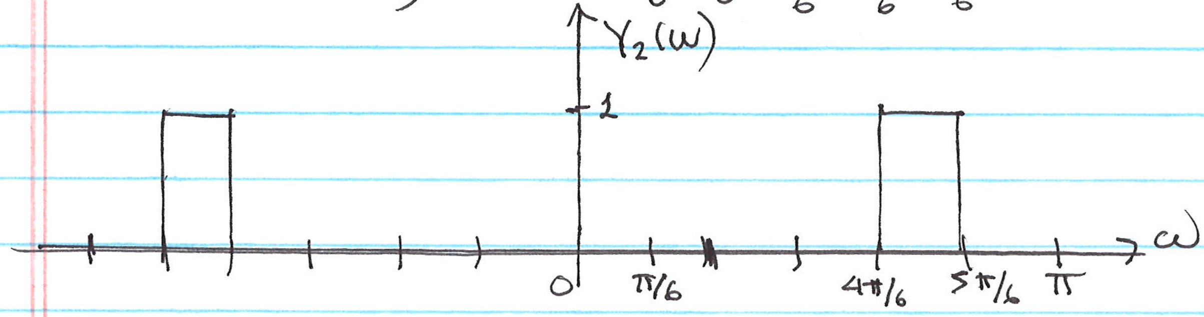
$\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$



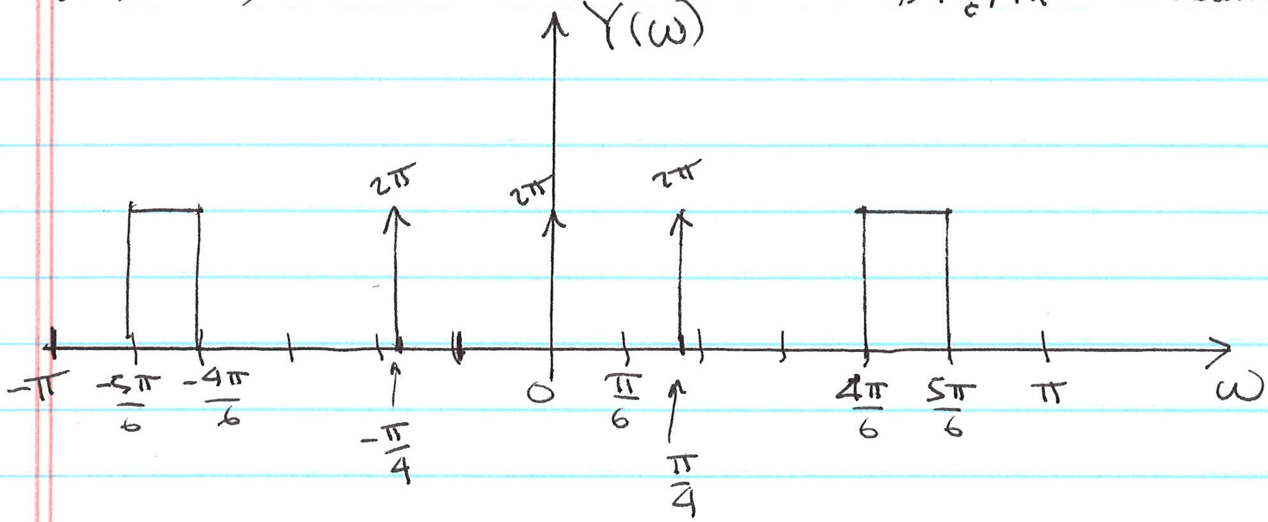




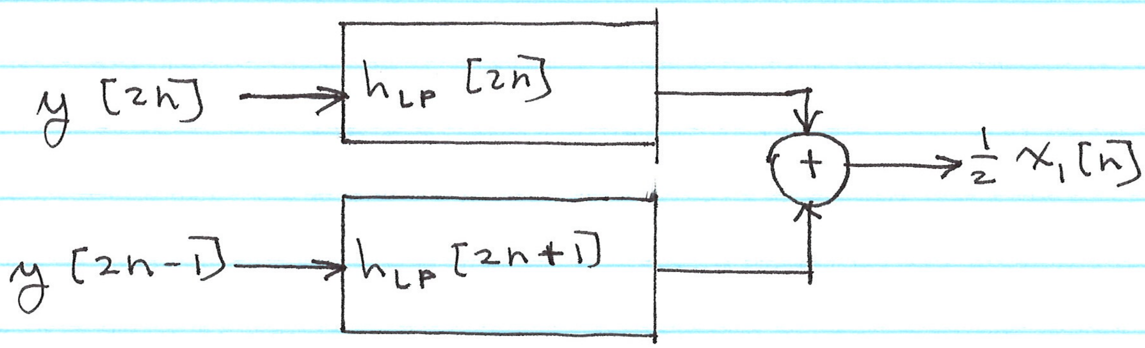


Digital Subbanding

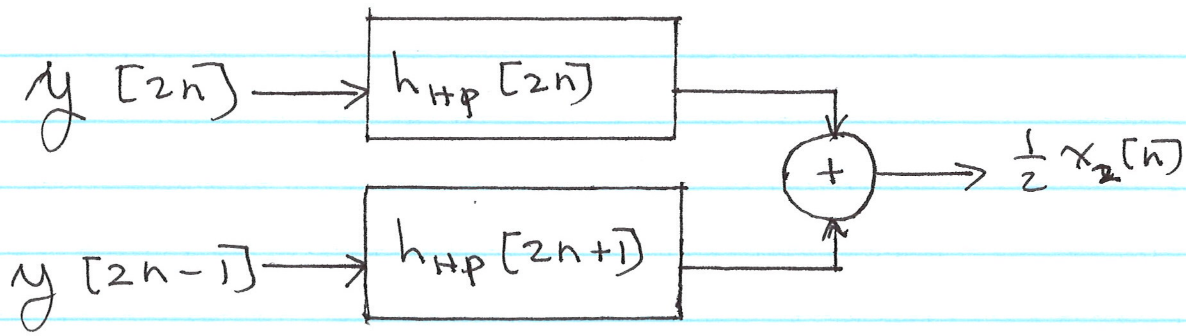
(2)-(a) THUS:



(b)

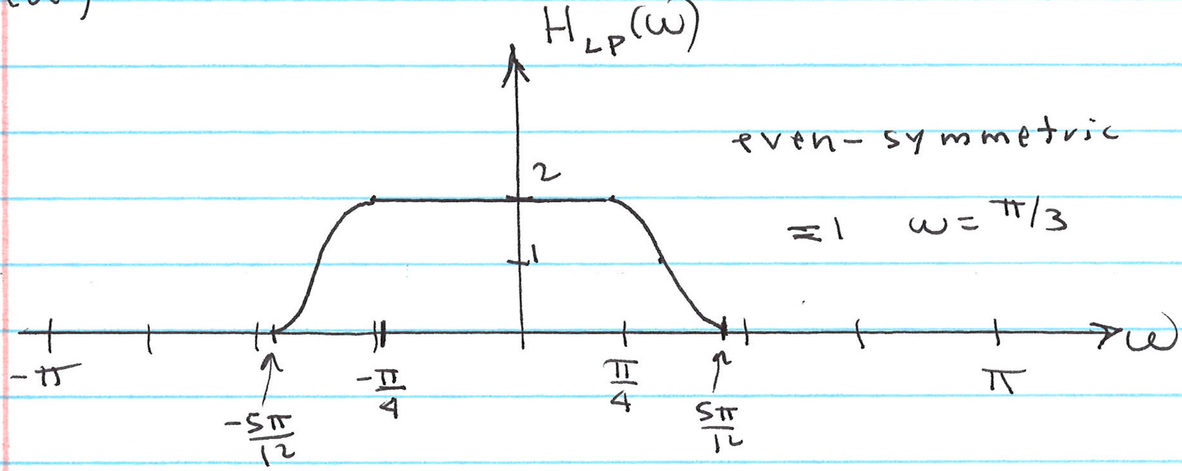


(c)



Problem 3 Soln Exam 2 Fall 2008

(a)



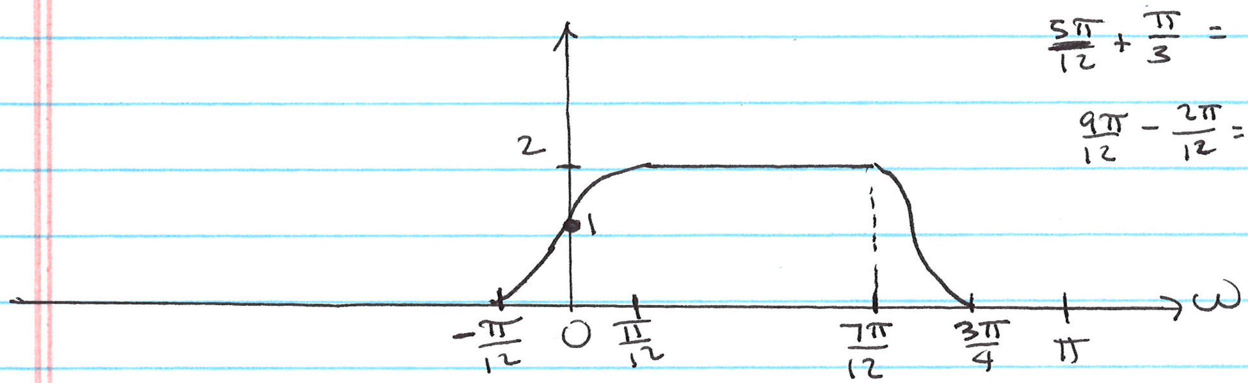
$$H(\omega) = H_{LP}\left(\omega - \frac{\pi}{3}\right)$$

note:

$$\frac{5\pi}{12} - \frac{3\pi}{12} = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$\frac{5\pi}{12} + \frac{\pi}{3} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

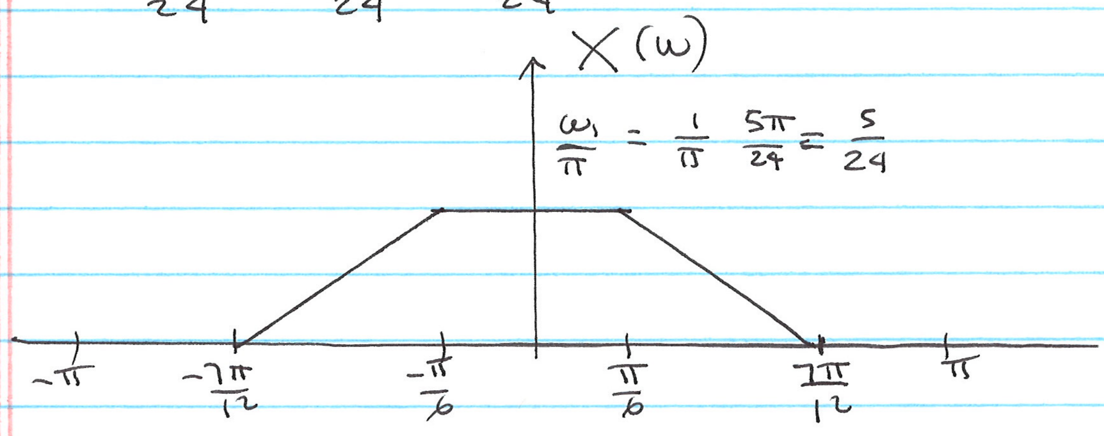
$$\frac{9\pi}{12} - \frac{2\pi}{12} = \frac{7\pi}{12}$$



(b)

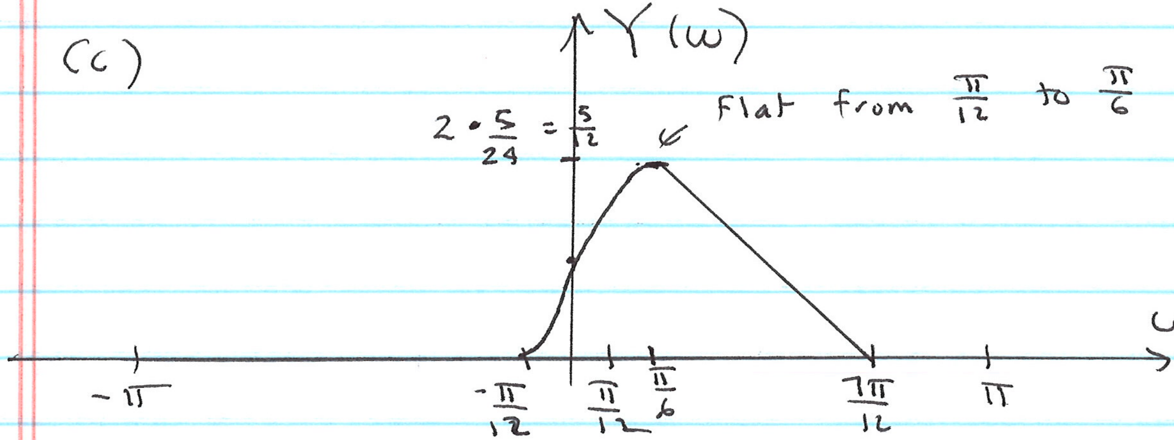
$$\frac{5\pi}{24} + \frac{9\pi}{24} = \frac{14\pi}{24} = \frac{7\pi}{12}$$

$$\frac{9\pi}{24} - \frac{5\pi}{24} = \frac{4\pi}{24} = \pi/6$$

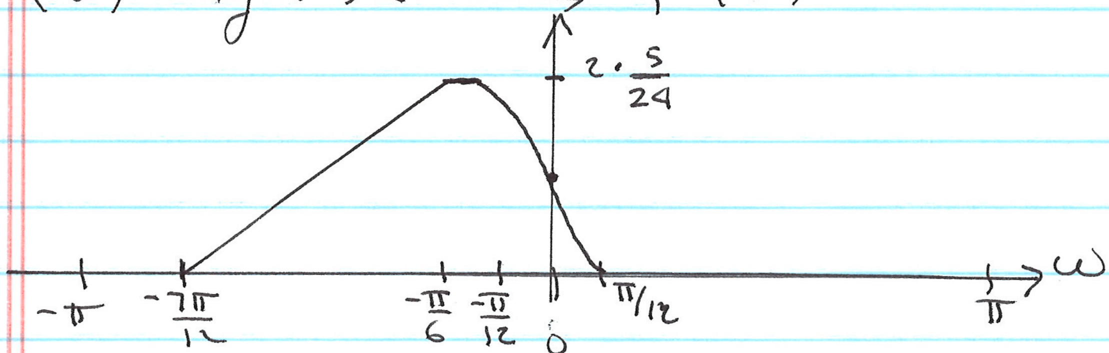


Prob. 3 Sol'n (cont.)

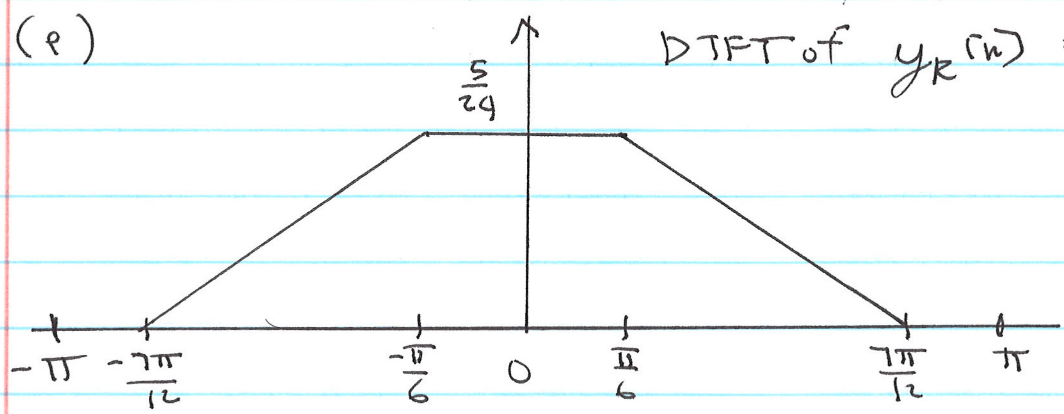
(c)



(d) $y^*[n] \overset{DTFT}{\longleftrightarrow} Y^*(-\omega)$



(e)



DTFT of $y_R[n]$ = sum of (c) & (d) ÷ 2

(f) Yes, $y_R[n] = x[n]$!

plot in (e) same as plot in (b)

Prob. 3 (g)

$$H(\omega) = H_{LP}\left(\omega - \frac{\pi}{3}\right) = 1 + \cos\left(6\left(\left|\omega - \frac{\pi}{3}\right| - \frac{\pi}{4}\right)\right)$$

for $-\frac{\pi}{12} < \omega < \frac{\pi}{12}$

Over $-\frac{\pi}{12} < \omega < \frac{\pi}{12}$, $\left|\omega - \frac{\pi}{3}\right| = \frac{\pi}{3} - \omega$

over $|\omega| < \frac{\pi}{12}$

$$H(\omega) = 1 + \cos\left(6\left(\frac{\pi}{3} - \omega - \frac{\pi}{4}\right)\right)$$

$$= 1 + \cos\left(6\left(\frac{\pi}{12} - \omega\right)\right)$$

$$= 1 + \cos\left(6\left(\omega - \frac{\pi}{12}\right)\right)$$

$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

since $\cos(-\theta) = \cos\theta$

$$H(-\omega) = 1 + \cos\left(6\left(-\omega - \frac{\pi}{12}\right)\right)$$

$$= 1 + \cos\left(6\left(\omega + \frac{\pi}{12}\right)\right)$$

$$= 1 + \cos\left(6\omega + \frac{\pi}{12}\right)$$

$$= 1 - \sin(6\omega)$$

whereas:

$$H(\omega) = 1 + \cos\left(6\left(\omega - \frac{\pi}{12}\right)\right)$$

$$= 1 + \cos\left(6\omega - \frac{\pi}{12}\right)$$

$$= 1 + \sin(6\omega)$$

Thus, over $|\omega| < \frac{\pi}{12}$, $H(\omega) + H(-\omega) = 2$

works!!