ECE 538 Digital Signal Processing I Exam 2 Fall 2008
24 Oct. 2008

Cover Sheet

Test Duration: 55 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done on loose blank 8.5" x 11" sheets (not provided.)
You do **not** have to return this test sheet.

Prob. No.	**Topic of Problem**	**Points**
1.	Digital Subbanding	30
2.	Efficient Upsampling and Downsampling	30
3.	Complex-Valued Filtering for Spectral Efficiency	40

Problem 1. [30 points]

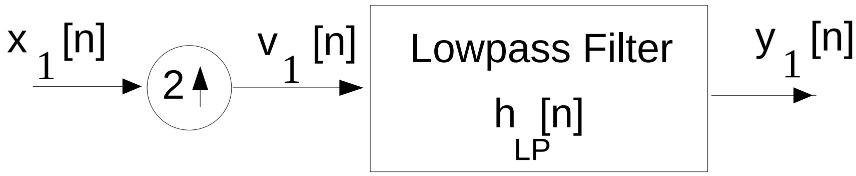

Figure 1.

Let $x_1[n]$ be a DT signal equal to a sum of sinwaves "turned on" for all time.

$$x_1[n] = 1 + (-j)^n + (j)^n$$

is input to the system above, where the impulse response of the lowpass filter is

$$h_{LP}[n] = 6\left\{\frac{\sin(\frac{\pi}{2}n)}{\pi n}\right\}\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}, \qquad -\infty < n < \infty,$$

(a) Plot the magnitude of the DTFT of the impulse response of the lowpass filter $h_{LP}[n]$, $H_{LP}(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible.

(b) Plot the magnitude of the DTFT of the output $y_1[n]$, $Y_1(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible. The respective frequencies of the sinusoidal components need to be clearly indicated.

(c) The up-sampling by a factor of 2 in Figure 1 above can be efficiently done via the top half of the block diagram in Figure 2 at the top of the next page.

   (i) Provide an analytical expression for $h_{LP}^{(0)}[n] = h_{LP}[2n]$ for $-\infty < n < \infty$. Simplify. Plot the magnitude of the DTFT of $h_{LP}^{(0)}[n]$, $|H_{LP}^{(0)}(\omega)|$, over $-\pi < \omega < \pi$.

   (ii) Is $y_1^{(0)}[n] = x_1[n]$? Explain your answer.

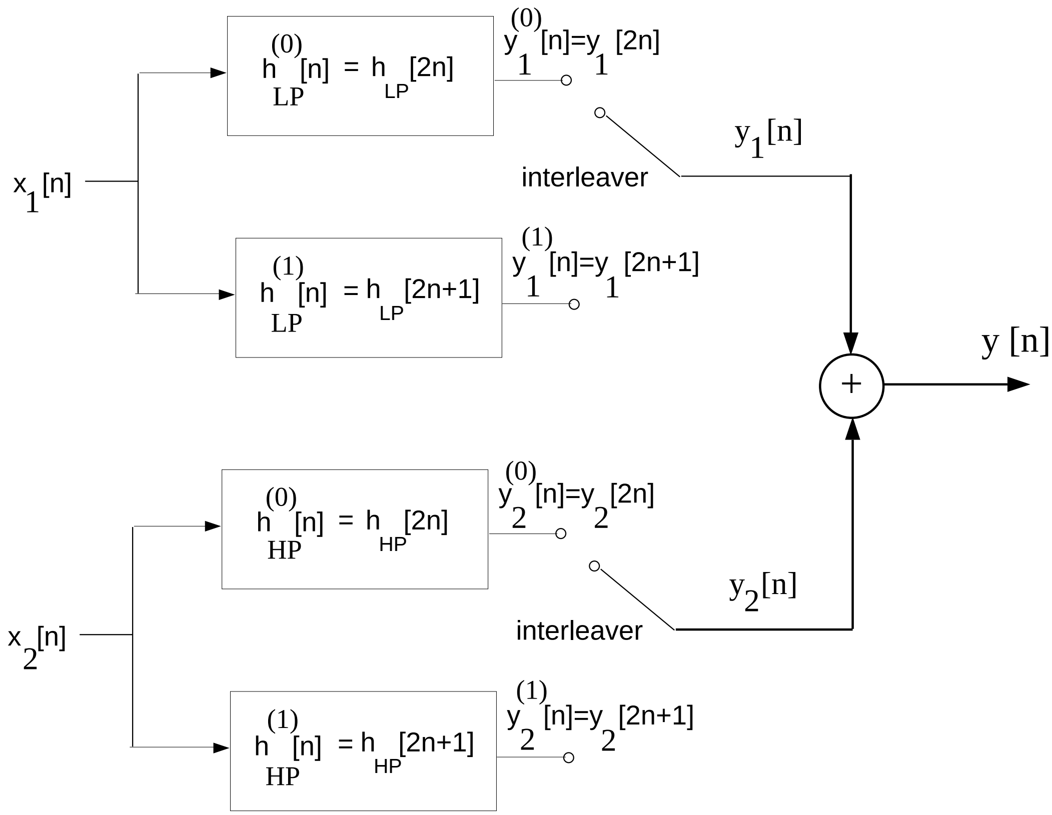

Figure 2.

Problem 2. [30 points] Let $x_1[n]$ be the DT signal described below:

$$x_2[n] = 2\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}\cos\left(\frac{\pi}{2}n\right)$$

We desire to frequency division multiplex $x_1[n]$ (from Problem 1) and $x_2[n]$ through digital subbanding as shown in Figure 2 above, where the highpass filter is

$$h_{HP}[n] = (-1)^n h_{LP}[n] = (-1)^n\, 6\left\{\frac{\sin(\frac{\pi}{2}n)}{\pi n}\right\}\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}, \qquad -\infty < n < \infty,$$

The sum signal is $y[n] = y_1[n] + y_2[n]$, where $y_1[n]$ is the same $y_1[n]$ created in Problem 1.

(a) Plot the magnitude of the DTFT of the sum signal $y[n] = y_1[n] + y_2[n]$. Show as much detail as possible. The respective frequencies of any sinusoidal components need to be clearly indicated as well as the edge frequencies of any rectangularly shaped spectral components.

(b) Draw a block diagram of a system to recover $x_1[n]$ from the sum signal $y[n] = y_1[n] + y_2[n]$. The recovery of $x_1[n]$ must be done in a computationally efficient manner. You CANNOT use any decimators and you CANNOT use any modulators (no multiplication by a sinewave). Clearly specify all the quantities in your block diagram.

(c) Draw a block diagram of a system to recover $x_2[n]$, from the sum signal $y[n] = y_1[n] + y_2[n]$. The same rules apply as those stated in part (b) above.

Problem 3. [40 points] This problem is about Vestigial Sideband (VSB) Modulation where we transmit a small part of the negative portion of the spectrum along with the positive frequency portion of the spectrum. *Note: all signals and filters in this problem have zero-phase in the frequency domain.*

(a) Consider the following complex-valued filter.

$$h[n] = e^{j\frac{\pi}{3}n} h_{LP}[n]$$

Let $H_{LP}(\omega)$ denote the DTFT (frequency response) of the filter $h_{LP}[n]$. $H_{LP}(\omega)$ is real-valued and even-symmetric. For $-\pi < \omega < \pi$, $H_{LP}(\omega)$ is mathematically described as

$$H_{LP}(\omega) = \begin{cases} 2, & |\omega| < \frac{\pi}{4} \\ 1 + \cos[6(|\omega| - \frac{\pi}{4})], & \frac{3\pi}{12} < |\omega| < \frac{5\pi}{12} \\ 0, & \frac{5\pi}{12} < |\omega| < \pi \end{cases}$$

Plot the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For illustrative purposes, consider the following input signal

$$x[n] = \frac{\sin\left(\frac{5\pi}{24}n\right)}{\pi n} \frac{\sin\left(\frac{9\pi}{24}n\right)}{\pi n}$$

Plot the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$.

(c) The signal in part (b) is run through the filter in part (a) to produce the output $y[n]$

$$y[n] = x[n] * h[n]$$

Plot the DTFT of $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

(d) The real-part of the complex-valued signal $y[n]$ may be expressed as

$$y_R[n] = Re\{y[n]\} = \frac{1}{2}\{y[n] + y^*[n]\}$$

Plot the DTFT of the complex-conjugate signal $y^*[n]$ over $-\pi < \omega < \pi$.

(e) Sum your respective answers to parts (c) and (d), and divide by 2, to form the DTFT of $y_R[n]$, denoted $Y_R(\omega)$. Plot $Y_R(\omega)$ over $-\pi < \omega < \pi$.

(f) Is your answer to part (e) equal to the DTFT of the original signal $x[n]$? That is, is $y_R[n] = x[n]$? Why or why not? Explain your answer.

(g) The output of the filter in part (a) includes some of the frequency content of $x[n]$ in the band $-\frac{\pi}{12} < \omega < 0$ in addition to the positive frequency portion of the signal in $0 < \omega < \pi$. Consider the more general case where the output of the filter $h[n]$ includes some of the frequency content of $x[n]$ in the band $-\Delta < \omega < 0$, in addition to the positive frequency portion of the signal in $0 < \omega < \pi$. In addition to the constraints $H(\omega) = 2$ over $\Delta < \omega < \pi$, and $H(\omega) = 0$ over $-\pi < \omega < -\Delta$, we showed that $H(\omega) + H(-\omega) = 2$ over $-\Delta < \omega < \Delta$ as well in order that the real part of the output $y[n] = x[n] * h[n]$ be equal to $x[n]$. Show that the filter $h[n]$ from part (a) satisfies this condition.