ECE 538 Digital Signal Processing I Exam 2 Fall 2007
30 Oct. 2007

Cover Sheet

Test Duration: 60 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Hilbert Transforms	30
2.	Digital Subbanding	40
3.	IIR Filter Design	30

Problem 1. [30 points]

(a) Plot the DTFT $X(\omega)$ of the signal $x[n]$ below over $-\pi < \omega < \pi$. Show as much detail as possible.

$$x[n] = 2\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\cos\left(\frac{3\pi}{8}n\right)$$

Write a simple, closed-form expression for $\hat{x}[n]$ equal to the Hilbert Transform of the signal $x[n]$.

(b) Let $x[n] = y[n]\cos(\omega_0 n)$, where $y[n]$ is a real-valued signal whose DTFT $Y(\omega)$ satisfies

$$Y(\omega) = 0 \quad \text{for} \quad \omega_c < |\omega| < \pi$$

Given that

$$\omega_0 - \omega_c > 0 \quad \text{and} \quad \omega_0 + \omega_c < \pi$$

Prove that the Hilbert Transform of $x[n]$ is

$$\hat{x}[n] = y[n]\sin(\omega_0 n)$$

(c) Plot the DTFT $X(\omega)$ of the signal $x[n]$ below over $-\pi < \omega < \pi$. Show as much detail as possible.

$$x[n] = 16\left\{\frac{\sin\left(\frac{3\pi}{8}n\right)}{\pi n}\,\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\right\}\cos\left(\frac{\pi}{2}n\right)$$

Write a simple, closed-form expression for $\hat{x}[n]$ equal to the Hilbert Transform of the signal $x[n]$.

Problem 2. [40 points] This problem is about Vestigial Sideband (VSB) Modulation where we transmit a small part of the negative portion of the spectrum along with the positive frequency portion of the spectrum. *Note: all signals and filters in this problem have zero-phase in the frequency domain.*

(a) Consider the following complex-valued filter.

$$h[n] = 16\, e^{j\frac{7\pi}{16}n} \left\{ \frac{\sin\left(\frac{2\pi}{16}n\right)}{\pi n} \; \frac{\sin\left(\frac{7\pi}{16}n\right)}{\pi n} \right\}$$

Plot the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For illustrative purposes, consider the following simple input signal

$$x[n] = \frac{\sin\left(\frac{3\pi}{4}n\right)}{\pi n}$$

Plot the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$.

(c) The signal in part (b) is run through the filter in part (a) to produce the output $y[n]$

$$y[n] = x[n] * h[n]$$

Plot the DTFT of $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

(d) The real-part of the complex-valued signal $y[n]$ may be expressed as

$$y_R[n] = Re\{y[n]\} = \frac{1}{2}\{y[n] + y^*[n]\}$$

Plot the DTFT of the complex-conjugate signal $y^*[n]$ over $-\pi < \omega < \pi$.

(e) Sum your respective answers to parts (c) and (d), and divide by 2, to form the DTFT of $y_R[n]$, denoted $Y_R(\omega)$. Plot $Y_R(\omega)$ over $-\pi < \omega < \pi$.

(f) Is your answer to part (e) equal to the DTFT of the original signal $x[n]$? That is, is $y_R[n] = x[n]$? Why or why not? Explain your answer.

(g) The output of the filter in part (a) includes some of the frequency content of $x[n]$ in the band $-\frac{\pi}{8} < \omega < 0$ in addition to the positive frequency portion of the signal in $0 < \omega < \pi$. Consider a more general case where the output of the filter $h[n]$ includes some of the frequency content of $x[n]$ in the band $-\Delta < \omega < 0$, in addition to the positive frequency portion of the signal in $0 < \omega < \pi$. In addition to the constraints $H(\omega) = 2$ over $\Delta < \omega < \pi$, and $H(\omega) = 0$ over $-\pi < \omega < -\Delta$, what condition does $H(\omega)$ have to satisfy over $-\Delta < \omega < \Delta$ in order that the real part of the output $y[n] = x[n] * h[n]$ be equal to $x[n]$?

Problem 3. [30 points]

A second-order digital filter is to be designed from an analog filter having two poles in the s-plane at $\frac{1}{5}(-3+4j) = -0.6+0.8j$ and $\frac{1}{5}(-3-4j) = -0.6-0.8j$ and two zeros at j and $-j$, via the bilinear transformation method characterized by the mapping

$$s = \frac{z-1}{z+1}$$

(a) Is the resulting digital filter (BIBO) stable? Briefly explain why or why not.

(b) Denote the frequency response of the resulting digital filter as $H(\omega)$ (the DTFT of its impulse response). You are given that in the range $0 < \omega < \pi$, there is only one value of ω for which $H(\omega) = 0$. Determine that value of ω.

(c) Draw a pole-zero diagram for the resulting **digital** filter. Give the exact locations of the poles and zeros of the digital filter in the z-plane.

(d) Plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. You are given that $H(0) = 8$. Be sure to indicate any frequency for which $|H(\omega)| = 0$. Also, specifically note the numerical value of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

(e) Determine the difference equation for the resulting digital filter.