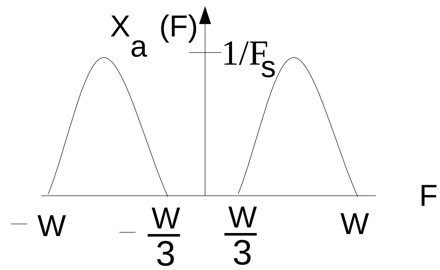
ECE 538 Digital Signal Processing I Exam 2 Fall 2006
Session 18, Live: 24 Oct. 2006

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **TWO** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Digital Subbanding	60
2.	Multi-Stage Upsampling/Interpolation	40

Problem 1. [60 points]



Consider the analog signal $x_a(t)$ whose Continuous Time Fourier Transform (CTFT), $X_a(F)$, is plotted above. $x_a(t)$, which is both real-valued and even-symmetric, is sampled at a rate of $F_s = \frac{16}{7}W$ to create the discrete-time signal $x[n] = x_a(n/F_s)$.

(a) Plot the magnitude and phase of the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|X(\omega)|$ and $\angle X(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

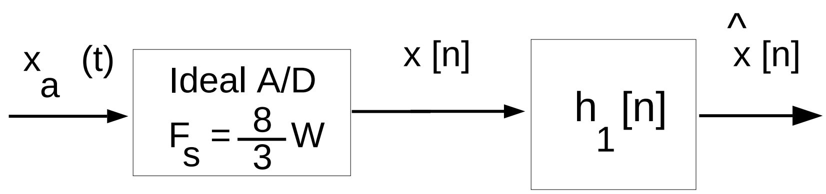


Figure 1. The sampling rate should be (16/7) W and not (8/3) W in the first box.

As shown in Figure 1, $x[n]$ is input to a discrete-time LTI system with impulse response

$$h_1[n] = 32\left\{\frac{\sin\left(\frac{\pi}{16}n\right)}{\pi n}\ \frac{\sin\left(\frac{7\pi}{16}n\right)}{\pi n}\right\}\sin\left(\frac{\pi}{2}n\right)$$

(b) Plot the magnitude and phase of the DTFT of $h_1[n]$, $H_1(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|H_1(\omega)|$ and $\angle H_1(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

A complex-valued signal is formed from $x[n]$ and the output $\hat{x}[n]$ in Figure 1 as

$$\tilde{x}[n] = x[n] + j\hat{x}[n]$$

(c) Plot the magnitude and phase of the DTFT of $\tilde{x}[n]$, $\tilde{X}(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|\tilde{X}(\omega)|$ and $\angle\tilde{X}(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

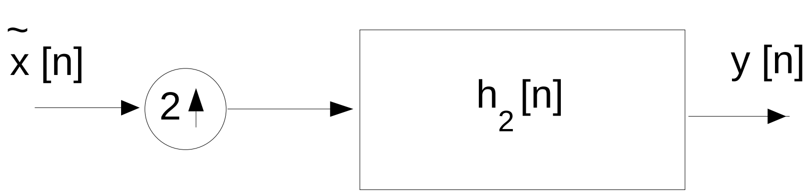


Figure 2.

For digital subbanding purposes, the signal $\tilde{x}[n]$ is input to the system in Figure 2 where the impulse response $h_2[n]$ is given by

$$h_2[n] = 16\,(-1)^n \left\{ \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \; \frac{\sin\left(\frac{\pi}{16}n\right)}{\pi n} \right\}$$

(d) Plot the magnitude and phase of the DTFT of the filter $h_2[n]$, $H_2(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|H_2(\omega)|$ and $\angle H_2(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

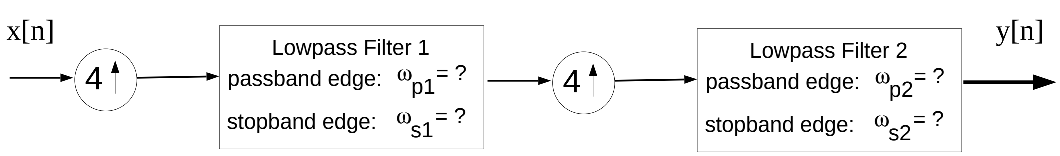
(e) Plot the magnitude and phase of the DTFT of the output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|Y(\omega)|$ and $\angle Y(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

(f) Draw a block diagram of a system for recovering $x[n]$ from $y[n]$. For this part, you don't have to worry about efficient computation. For this part and part (g), assume that four signals have been frequency divisioned multiplexed via digital subbanding, so that a linear combination of $h_2[n]$ and its Hilbert Transform $\hat{h}_2[n]$ is needed as a front-end filter to extract the sub-band that the signal derived from $x[n]$ occupies.

(g) Draw a block diagram of a system for recovering $x[n]$ from $y[n]$. For this part, you DO have to worry about efficiency. You are not allowed to use any decimators. You can ONLY use LTI systems where all inputs and impulse responses are real-valued.

Problem 2. [40 points]

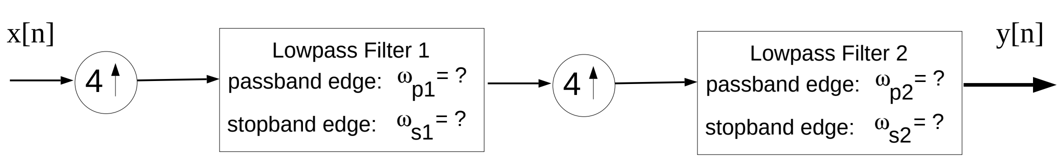
The signal $x[n] = x_a(n/F_s)$ is obtained by sampling an analog signal $x_a(t)$ having a bandwidth of W at a rate of $F_s = \frac{8}{3}W$. It is desired to increase the sampling rate by a factor of $L = 6$ in two stages via the system below, where ideally $y[n] = x_a(n/16W)$



(a) Determine the passband edge, ω_{p1}, and stopband edge, ω_{s1}, of the first lowpass filter.

(b) Determine the passband edge, ω_{p2}, and stopband edge, ω_{s2}, of the second lowpass filter.

Alternatively, we may achieve the same new higher sampling rate via the following system



(c) Determine the passband edge, ω_{p1}, and stopband edge, ω_{s1}, of the first lowpass filter.

(d) Determine the passband edge, ω_{p2}, and stopband edge, ω_{s2}, of the second lowpass filter.