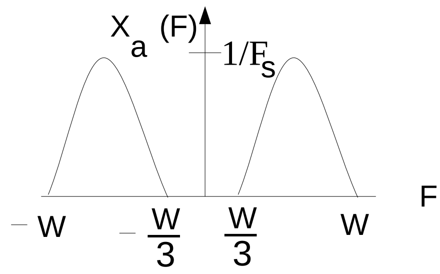
ECE 538 Digital Signal Processing I Exam 2 Fall 2005
26 Oct. 2005

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Digital Subbanding	50
2.	Multi-Stage Upsampling/Interpolation	20
3.	IIR Filter Design	30

Problem 1. [50 points]



Consider the analog signal $x_a(t)$ whose Continuous Time Fourier Transform (CTFT), $X_a(F)$, is plotted above. $x_a(t)$, which is both real-valued and even-symmetric, is sampled at a rate of $F_s = \frac{8}{3}W$ to create the discrete-time signal $x[n] = x_a(n/F_s)$.

(a) Plot the magnitude and phase of the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|X(\omega)|$ and $\angle X(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

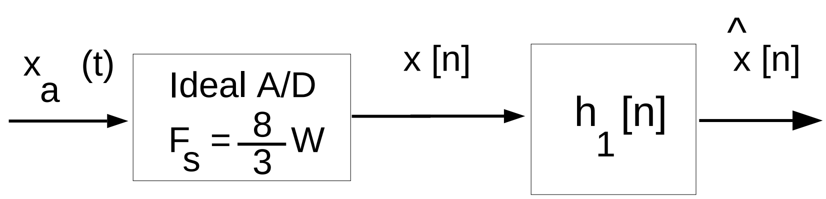


Figure 1.

As shown in Figure 1, $x[n]$ is input to a discrete-time LTI system with impulse response

$$h_1[n] = 16\left\{\frac{\sin\left(\frac{3\pi}{8}n\right)}{\pi n}\,\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\right\}\sin\left(\frac{\pi}{2}n\right)$$

(b) Plot the magnitude and phase of the DTFT of $h_1[n]$, $H_1(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|H_1(\omega)|$ and $\angle H_1(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

A complex-valued signal is formed from $x[n]$ and the output $\hat{x}[n]$ in Figure 1 as

$$\tilde{x}[n] = x[n] + j\hat{x}[n]$$

(c) Plot the magnitude and phase of the DTFT of $\tilde{x}[n]$, $\tilde{X}(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|\tilde{X}(\omega)|$ and $\angle\tilde{X}(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

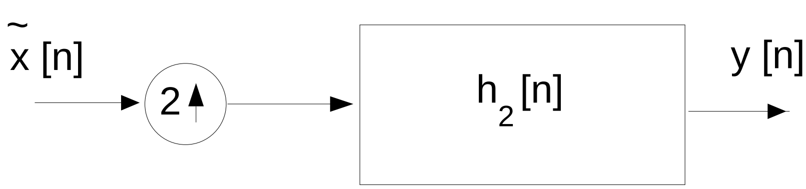


Figure 2.

For digital subbanding purposes, the signal $\tilde{x}[n]$ is input to the system in Figure 2 where the impulse response $h_2[n]$ is given by

$$h_2[n] = 8\,(-1)^n \left\{ \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \; \frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n} \right\}$$

(d) Plot the magnitude and phase of the DTFT of the filter $h_2[n]$, $H_2(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|H_2(\omega)|$ and $\angle H_2(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

(e) Plot the magnitude and phase of the DTFT of the output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|Y(\omega)|$ and $\angle Y(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

(f) Draw a block diagram of a system for recovering $x[n]$ from $y[n]$. For this part, you don't have to worry about efficient computation. Assume that we did similar operations to other signals to put them in subbands different from the subband that $y[n]$ occupies.

(g) Draw a block diagram of a system for recovering $x[n]$ from $y[n]$. For this part, you DO have to worry about efficiency. You are not allowed to use any decimators. You can ONLY use LTI systems where all inputs and impulse responses are real-valued.

Problem 2. [20 points]
GIVEN NOBLE's FIRST IDENTITY TO USE IN PROBLEM 2. If $E(\omega)$ in Figure 3(b) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). You are GIVEN this as Noble's First Identity to use in Problem 2 below.

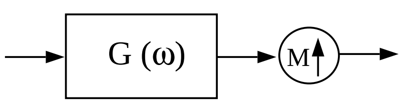


Figure 3(a).

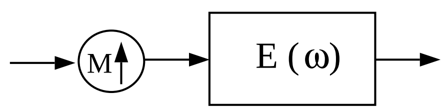


Figure 3(b).

(a) Determine the impulse response $h_a[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). Plot the magnitude of the DTFT of $h_a[n]$ over $-\pi < \omega < \pi$.

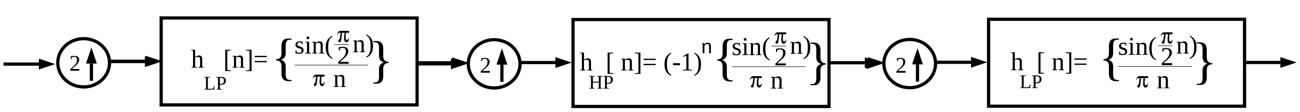


Figure 4(a).

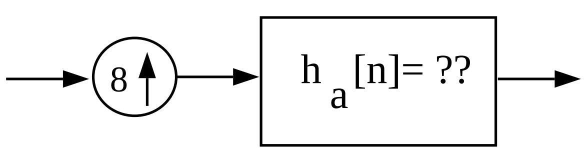


Figure 4(b).

Problem 3. [30 points]

A second-order digital filter is to be designed from an analog filter having two poles in the s-plane at $-0.2+0.4j$ and $-0.2-0.4j$ and two zeros at $j\sqrt{3}$ and $-j\sqrt{3}$, via the bilinear transformation method characterized by the mapping

$$s = \frac{z-1}{z+1}$$

Note that $-0.2+0.4j = (1/5) + j(2/5)$ and $j\sqrt{3} = j\tan(\pi/3)$

(a) Is the resulting digital filter (BIBO) stable? Briefly explain why or why not.

(b) Denote the frequency response of the resulting digital filter as $H(\omega)$ (the DTFT of its impulse response). You are given that in the range $0 < \omega < \pi$, there is only one value of ω for which $H(\omega) = 0$. Determine that value of ω.

(c) Draw a pole-zero diagram for the resulting **digital** filter. Give the exact locations of the poles and zeros of the digital filter in the z-plane.

(d) Plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. You are given that $H(0) = 6$. Be sure to indicate any frequency for which $|H(\omega)| = 0$. Also, specifically note the numerical value of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

(e) Determine the difference equation for the resulting digital filter.