ECE 538 Digital Signal Processing I Exam 2 Fall 2004
Session 21 Live: 13 Oct. 2004

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	Topic of Problem	Points
1.	Digital Upsampling	35
2.	Digital Subbanding	30
3.	Multi-Stage Upsampling/Interpolation	35

Problem 1. [35 points]

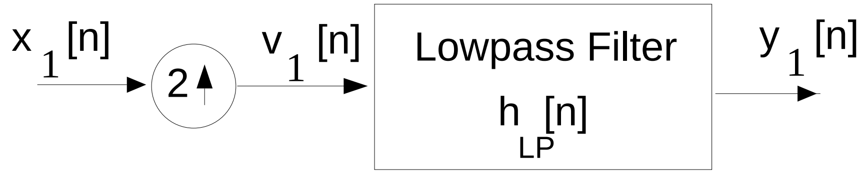

Figure 1.

The DT signal

$$x_1[n] = 2\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}\cos\left(\frac{\pi}{2}n\right)$$

is input to the system above, where the impulse response of the lowpass filter is

$$h_{LP}[n] = 6\left\{\frac{\sin(\frac{\pi}{2}n)}{\pi n}\right\}\left\{\frac{\sin(\frac{\pi}{6}n)}{\pi n}\right\}, \qquad -\infty < n < \infty,$$

The zero inserts may be mathematically described as $v_1[n] = \sum_{k=-\infty}^{\infty} x_1[k]\delta[n-2k]$.

(a) Plot the magnitude of the DTFT of the impulse response of the lowpass filter $h_{LP}[n]$, $H_{LP}(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible. This lowpass filter has a "don't care" region where the gain rolls off linearly from 1 to 0. Does this have any impact on the upsampling process?

(b) Plot the magnitude of the DTFT of the output $y_1[n]$, $Y_1(\omega)$, over $-\pi < \omega < \pi$. Show as much detail as possible. The edge frequencies of any rectangularly shaped spectral components need to be clearly indicated.

(c) The up-sampling by a factor of 2 in Figure 1 above can be efficiently done via the top half of the block diagram in Figure 2 at the top of the next page.

(i) Provide an analytical expression for $h_{LP}^{(0)}[n] = h_{LP}[2n]$ for $-\infty < n < \infty$. Simplify. Plot the magnitude of the DTFT of $h_{LP}^{(0)}[n]$, $|H_{LP}^{(0)}(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Is $y_1^{(0)}[n] = x_1[n]$? Explain your answer.

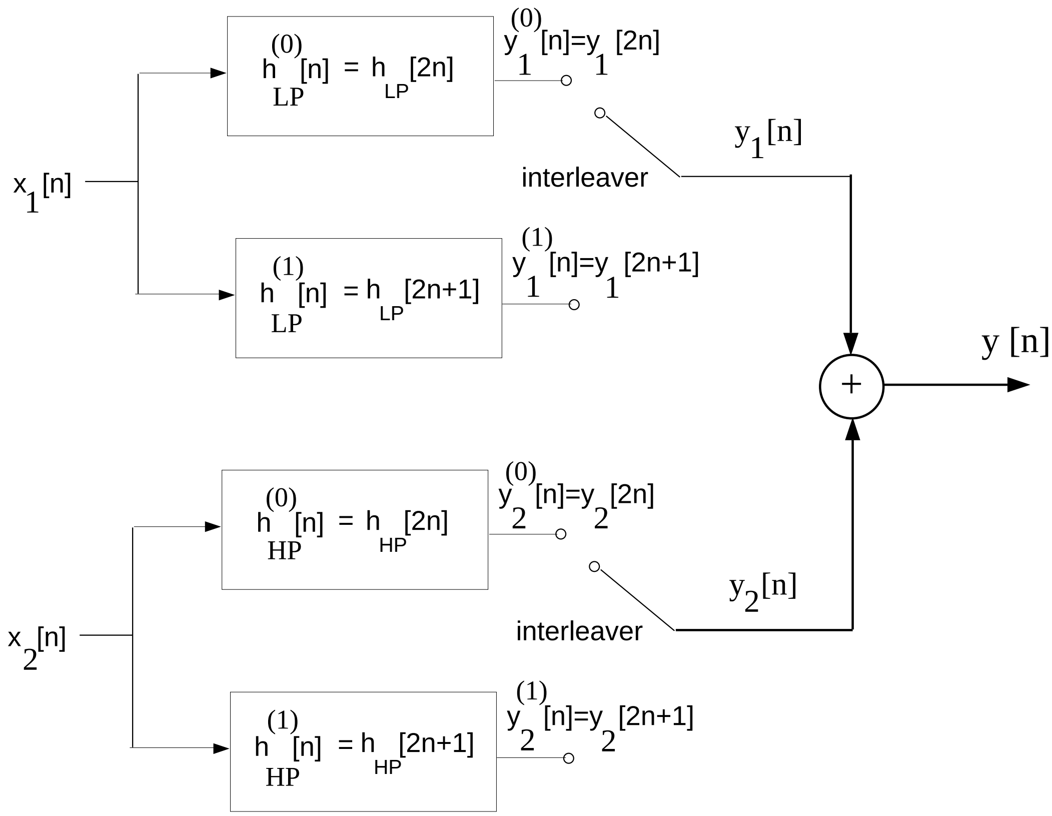

Figure 2.

Problem 2. [30 points] Let $x_2[n]$ be a DT signal equal to a sum of sinwaves "turned on" for all time.

$$x_2[n] = 1 + (-j)^n + (j)^n$$

We desire to frequency division multiplex $x_1[n]$ (from Problem 1) and $x_2[n]$ through digital subbanding as shown in Figure 2 above, where the highpass filter is

$$h_{HP}[n] = (-1)^n h_{LP}[n] = (-1)^n \, 6 \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \left\{ \frac{\sin(\frac{\pi}{6}n)}{\pi n} \right\}, \qquad -\infty < n < \infty,$$

The sum signal is $y[n] = y_1[n] + y_2[n]$, where $y_1[n]$ is the same $y_1[n]$ created in Problem 1.

(a) Plot the magnitude of the DTFT of the sum signal $y[n] = y_1[n] + y_2[n]$. Show as much detail as possible. The respective frequencies of any sinusoidal components need to be clearly indicated as well as the edge frequencies of any rectangularly shaped spectral components.

(b) Draw a block diagram of a system to recover $x_1[n]$ from the sum signal $y[n] = y_1[n] + y_2[n]$. The recovery of $x_1[n]$ must be done in a computationally efficient manner. You CANNOT use any decimators and you CANNOT use any modulators (no multiplication by a sinewave). Clearly specify all the quantities in your block diagram.

(c) Draw a block diagram of a system to recover $x_2[n]$, from the sum signal $y[n] = y_1[n] + y_2[n]$. The same rules apply as those stated in part (b) above.

If $E(\omega)$ in Figure 3(b) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). You are GIVEN this as Noble's First Identity to use in Problem 3 below.

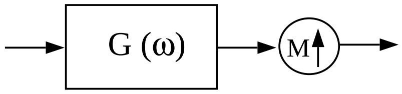

Figure 3(a).

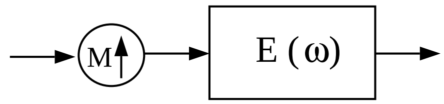

Figure 3(b).

Problem 3. [35 points]

(a) Determine the impulse response $h_a[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). Plot the magnitude of the DTFT of $h_a[n]$ over $-\pi < \omega < \pi$.

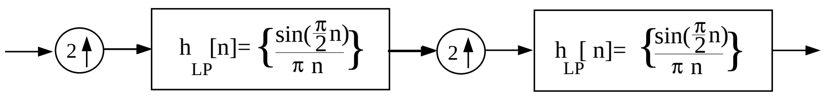

Figure 4(a).

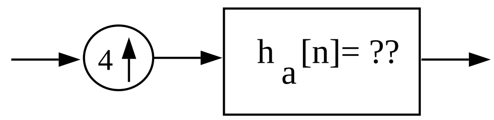

Figure 4(b).

(b) Determine the impulse response $h_b[n]$ in Figure 5(b) so that the I/O relationship of the system in Figure 5(b) is exactly the same as the I/O relationship of the system in Figure 5(a). Plot the magnitude of the DTFT of $h_b[n]$ over $-\pi < \omega < \pi$.

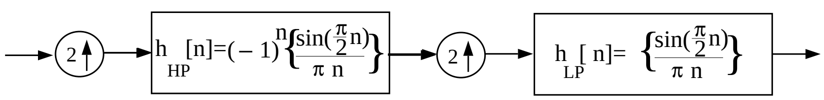

Figure 5(a).

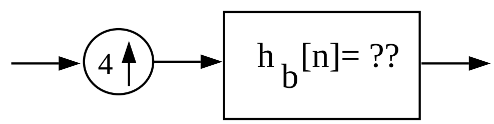

Figure 5(b).

(c) Determine the impulse response $h_c[n]$ in Figure 6(b) so that the I/O relationship of the system in Figure 6(b) is exactly the same as the I/O relationship of the system in Figure 6(a). Plot the magnitude of the DTFT of $h_c[n]$ over $-\pi < \omega < \pi$.

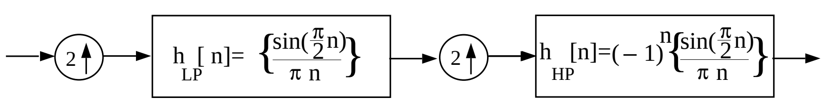

Figure 6(a).

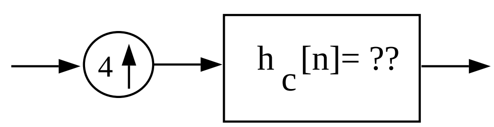

Figure 6(b).

(d) Determine the impulse response $h_d[n]$ in Figure 7(b) so that the I/O relationship of the system in Figure 7(b) is exactly the same as the I/O relationship of the system in Figure 7(a). Plot the magnitude of the DTFT of $h_d[n]$ over $-\pi < \omega < \pi$.

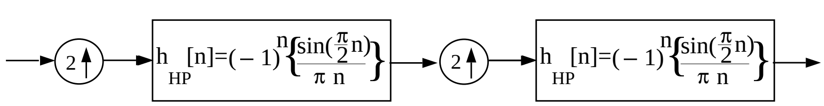

Figure 7(a).

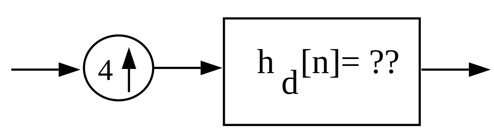

Figure 7(b).