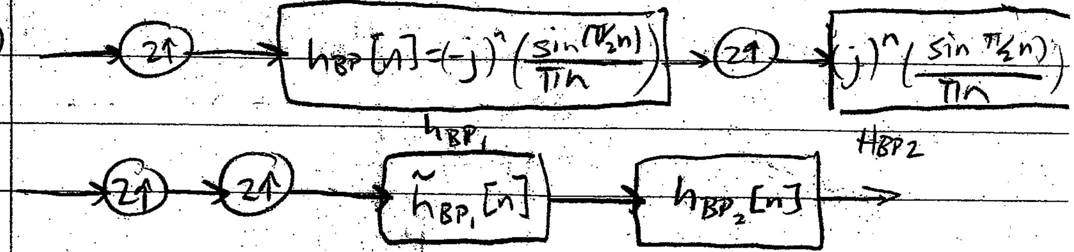
Exam 2 F '03 Soln

Sol'n to Prob. 1 (a)

a)



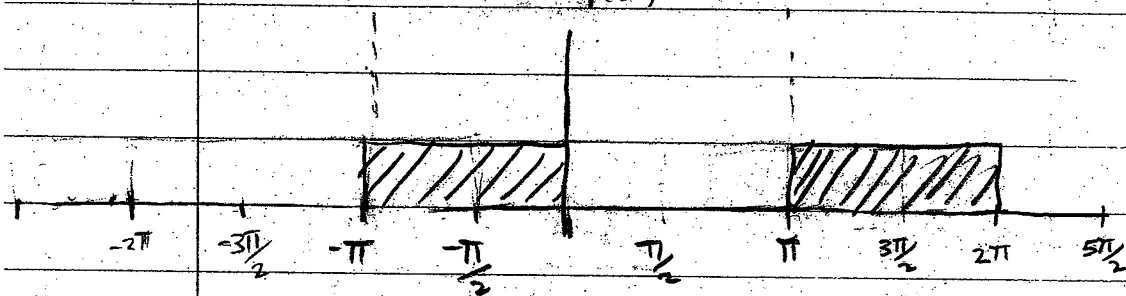
$$\tilde{H}_{BP_1}(\omega) = H_{BP_1}(2\omega)$$

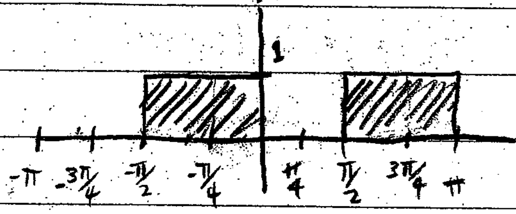
$$h_{BP}[n] = (e^{-j\pi/2})^n \left(\frac{\sin \frac{\pi}{2} n}{\pi n}\right)$$

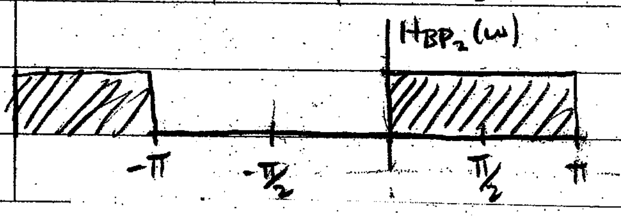
$\pi + \pi/2$

2π

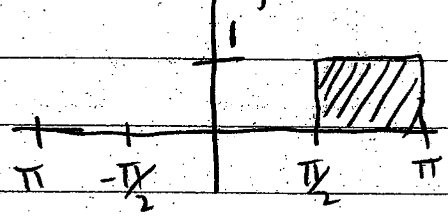
$\frac{3\pi}{2} + \pi/2 \quad \frac{4\pi}{2} + \frac{\pi}{2}$



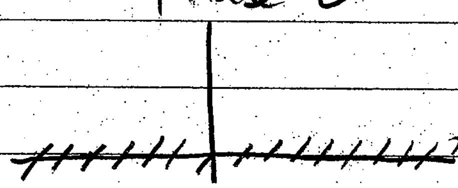




$$\tilde{h}_{BP_1}[n] * h_{BP_2}[n] = \tilde{H}_{BP_1}(2\omega) \cdot H_{BP_2}(\omega)$$



Phase = 0



Sol'n to Prob. 1(b)

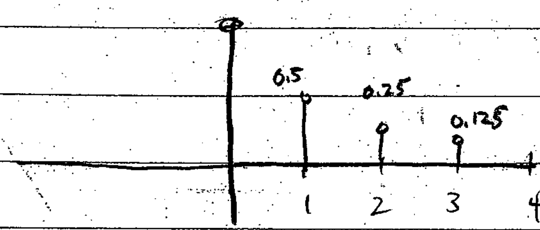
b) $\rightarrow (2\uparrow) \rightarrow \boxed{h_1[n]} \rightarrow (2\uparrow) \rightarrow \boxed{h_2[n]}$

$\rightarrow (2\uparrow) \rightarrow (2\uparrow) \rightarrow \boxed{\tilde{h}_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow$

$$\tilde{H}_1(\omega) = H_1(2\omega) =$$

$$H_1(\omega) = \frac{1}{1 - 0.5e^{-j\omega}}$$

$h_1[n]$



$$h_1[n] \approx \{1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16}, \tfrac{1}{32}, \ldots 0\}$$

$$h_{1up}[n] = \sum_{k=-\infty}^{\infty} h_1[k]\,\delta[n-2k] = \{\underset{\substack{\uparrow\\0}}{1}, 0, \tfrac{1}{2}, 0, \tfrac{1}{4}, 0, \tfrac{1}{8}, 0, \tfrac{1}{16}, 0, \ldots$$

$$h_2[n] = \{\underset{\uparrow}{2}, 1\}$$

$$heq[n] = \{2, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{8}, \ldots\}$$

$$= h_{1up}[n] * h_2[n]$$

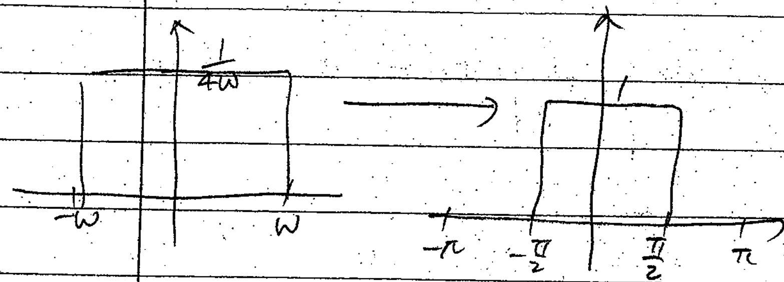
$$= 2h_{1up}[n] + h_{1up}[n-1]$$

since $\quad h_2[n] = 2\delta[n] + \delta[n-1]$

Prob 2

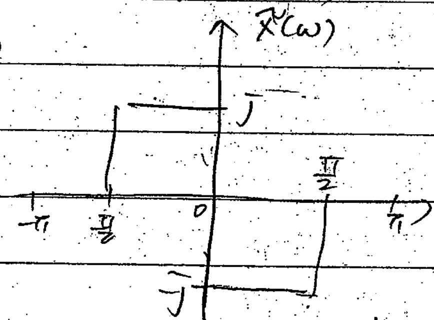
2. (a) solve)

① $X_a(F)$ $\qquad$ $X(\omega)$

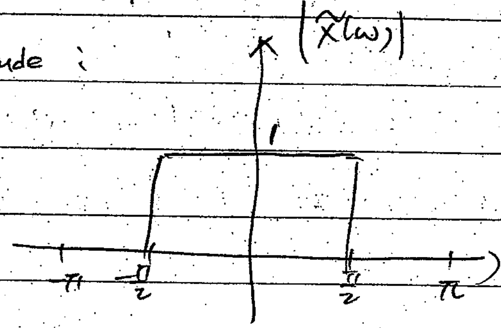


② $\tilde{X}(n) = x(n) * h(n)$ where $h(n)$ = Impulse response of Hilbert Transform

$\therefore \tilde{X}(\omega) = X(\omega) H(\omega)$



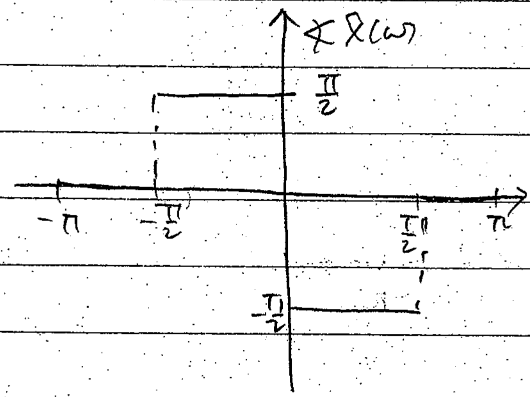
i) magnitude :



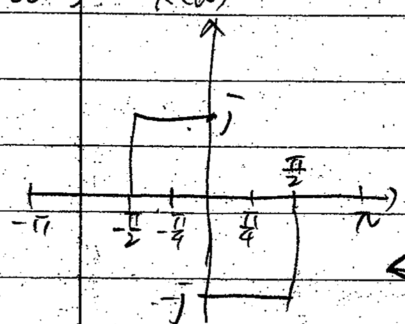
Prob. 2 Soln (cont)

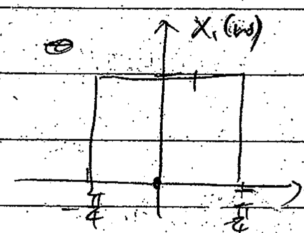
ii) phase : for $-\frac{\pi}{2} < \omega < 0 \Rightarrow \measuredangle X(\omega) = \frac{\pi}{2}$

for $0 < \omega < \frac{\pi}{2} \Rightarrow \measuredangle X(\omega) = -\frac{\pi}{2}$



(b) solve





with $x_1(n) = \frac{\sin\frac{\pi}{4}n}{\pi n}$

$\tilde{X}(\omega) = j X_1(\omega + \frac{\pi}{4}) - j X_1(\omega - \frac{\pi}{4})$

$$\tilde{X}(\omega) = j X_1(\omega + \tfrac{\pi}{4}) - j X_1(\omega - \tfrac{\pi}{4})$$
$$= j\left[X_1(\omega + \tfrac{\pi}{4}) - X_1(\omega - \tfrac{\pi}{4})\right]$$

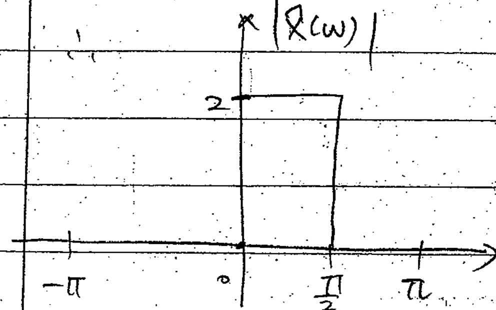
IDTFT)
$$\tilde{x}(n) = j\left(x_1(n) e^{-j\frac{\pi}{4}n} - x_1(n) e^{+j\frac{\pi}{4}n}\right)$$
$$= j x_1(n) \underbrace{\left[e^{-j\frac{\pi}{4}n} - e^{+j\frac{\pi}{4}n}\right]}_{-2j\sin\frac{\pi}{4}n} = 2\sin\tfrac{\pi}{4}n \cdot x_1(n)$$

$\therefore \quad \tilde{x}(n) = \frac{2\sin^2\frac{\pi}{4}n}{\pi n}$

Prob 2 Sol'n (cont.)

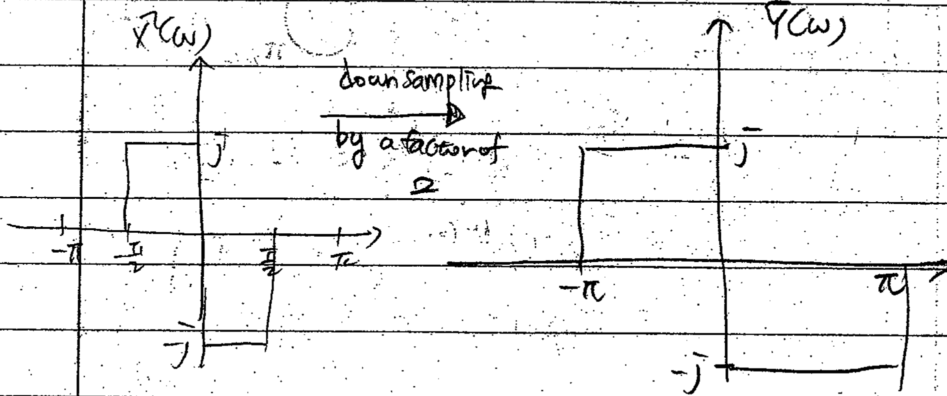
(c). solve) $\hat{x}(n) = x(n) + j\tilde{x}(n)$

: blanking out negative frequency of $X(n)$



(d) solve) $\tilde{y}(n) = \tilde{x}(2n)$

① $\Rightarrow$ $\tilde{Y}(\omega) = \frac{1}{2}\tilde{X}(\frac{\omega}{2}) + \frac{1}{2}\tilde{X}(\frac{\omega - 2\pi}{2})$

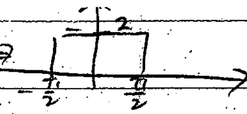


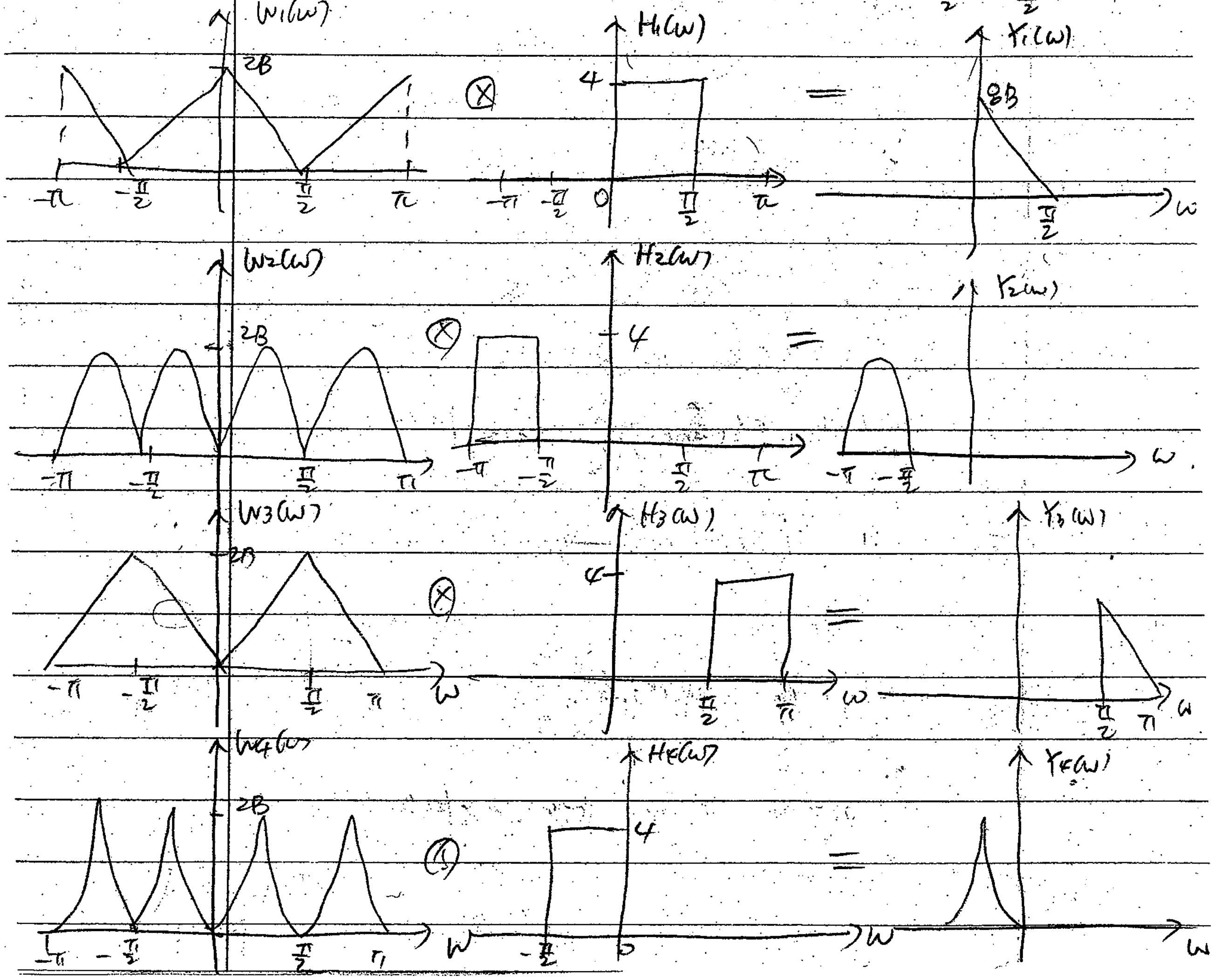
② $\hat{y}(n) = \delta(n) + j\tilde{y}(n)$.

$\xrightarrow{DTFT}$ $\hat{Y}(\omega) = 1 + j\tilde{Y}(\omega) = \begin{cases} 2 & 0 < \omega < \pi \\ 0 & -\pi < \omega < 0 \end{cases}$

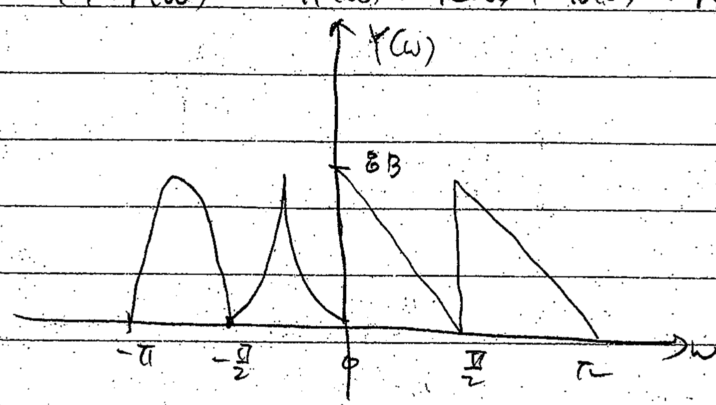
Prob. 3. (a) solve)

$h_{LP}(n) = 2 \cdot \frac{\sin\frac{\pi}{2}n}{\pi n} \Rightarrow H_{LP}(\omega) \Rightarrow$



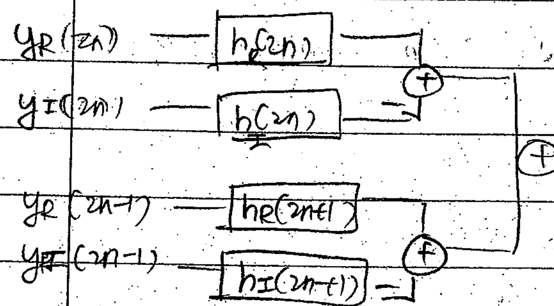


$\therefore Y(\omega) = Y_1(\omega) + Y_2(\omega) + Y_3(\omega) + Y_4(\omega)$



$a = \mathrm{Re}\{(y_R(n) + j\,y_I(n)) * (h_R(n) + j\,h_I(n))\} = y_R(n)\,h_R(n) - y_I(n)\,h_I(n)$

~~III~~)



~~IIII~~) apply this!

i) for $x_1(n)$ $x_2(n)$, we can use the same system

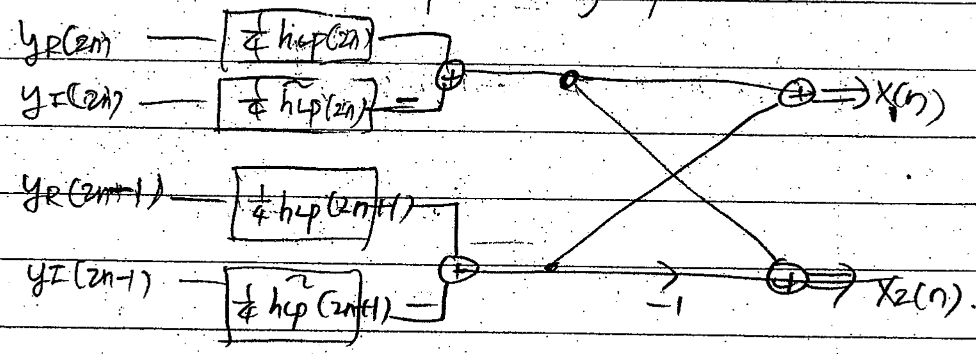
$\because$ $h_1(n) = h_{LP}(n) + j\,\tilde{h}_{LP}(n)$

$h_2(n) = e^{j\pi n}(h_{LP}(n) + j\,\tilde{h}_{LP}(n))$

$$\Rightarrow \begin{cases} h_1(2n) = h_{LP}(2n) + j\,\tilde{h}_{LP}(2n) \\ h_2(2n) = e^{j2\pi n}(h_{LP}(2n) + j\,\tilde{h}_{LP}(2n)) \\ \qquad\;\; = h_{LP}(2n) + j\,\tilde{h}_{LP}(2n) \end{cases}$$

$$\begin{cases} h_1(2n+1) = h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1) \\ h_2(2n+1) = e^{j\pi(2n+1)}(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)) \\ \qquad\qquad\; = e^{j\pi}(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)) \\ \qquad\qquad\; = -(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)) \end{cases}$$

~~IIA~~)



Prob. 3 Sol'n.

$y_1(n) = x_{a1}\left(\frac{n}{4B}\right) + j\,\tilde{x}_{a1}\left(\frac{n}{4B}\right)$

$y_2(n) = \left(x_{a2}\left(\frac{n}{4B}\right) + j\,\tilde{x}_{a2}\left(\frac{n}{4B}\right)\right)\cdot e^{-j\pi n}$

$y_3(n) = \left(x_{a3}\left(\frac{n}{4B}\right) - j\,\tilde{x}_{a3}\left(\frac{n}{4B}\right)\right)\cdot e^{j\pi n}$

$y_4(n) = \left(x_{a4}\left(\frac{n}{4B}\right) - j\,\tilde{x}_{a4}\left(\frac{n}{4B}\right)\right)$

$\Rightarrow\; y(n) - \boxed{\tfrac{1}{4} h_i(n)} \xrightarrow{y_i(n)} (\downarrow 2) - \{\mathrm{Re}\} - x_i(n)$

$\Rightarrow$ in an efficient way. by using efficient downsampling

because $H_i(\omega)$ has gain $= 4$

ii) For $x_3(n)$ and $x_4(n)$, we can use ~~the~~ same system

$\because\; h_3(n) = e^{j\frac{\pi}{2}n}\, h_1(n)$

$h_4(n) = e^{-j\frac{\pi}{2}n}\, h_1(n)$

$h_3(2n) = e^{j\pi n}\left(h_{LP}(2n) + j\,\tilde{h}_{LP}(2n)\right)$

$h_4(n) = e^{j\pi n}\left(h_{LP}(2n) + j\,\tilde{h}_{LP}(2n)\right)$

$h_3(2n+1) = e^{j\pi n}\cdot e^{j\frac{\pi}{2}}\left(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)\right)$

$= e^{j\pi n}\, j\left(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)\right)$

$= e^{j\pi n}\left(-\tilde{h}_{LP}(2n+1) + j\,h_{LP}(2n+1)\right)$

$h_4(2n+1) = e^{j\pi n}\cdot e^{-j\frac{\pi}{2}}\left(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)\right)$

$= e^{j\pi n}\cdot -j\left(h_{LP}(2n+1) + j\,\tilde{h}_{LP}(2n+1)\right)$

$= e^{j\pi n}\left(\tilde{h}_{LP}(2n+1) - j\,h_{LP}(2n+1)\right)$

$;\; e^{j\pi n} = e^{-j\pi n} = (-1)^n$

$\Rightarrow$