EE538 Digital Signal Processing I — Session 25
Exam 2 — Live: Fri., Oct. 24, 2002

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	Principles of Upsampling & Downsampling (Time & Frequency Domain Analysis) with Multiple Stages	30
2.	Hilbert Transformers; Complex Analytic Signals	30
3.	Digital Subbanding; Efficient Downsampling	40

Problem 1. [30 points]
GIVEN NOBLE's IDENTITIES TO USE IN PROBLEM 1.

(a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

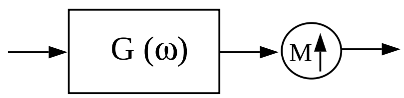

Figure 1(a).

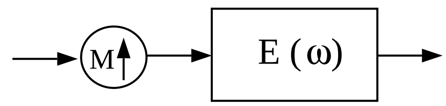

Figure 1(b).

(b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

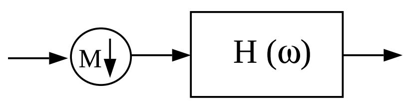

Figure 2(a).

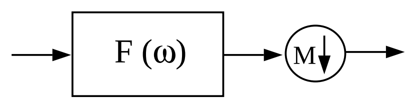

Figure 2(b).

Problem 1. [20 points]

(a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.

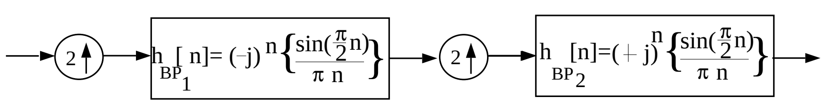

Figure 3(a).

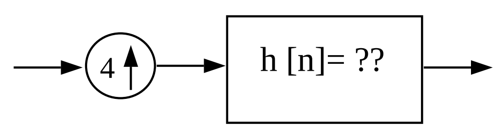

Figure 3(b).

(b) Determine the numerical values of the impulse response $h_{\text{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). Analyze system of Figure 4(a) in time domain using Noble's First Identity. You must simplify your answer as much as possible for full credit.

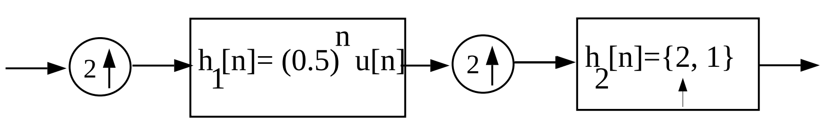

Figure 4(a).

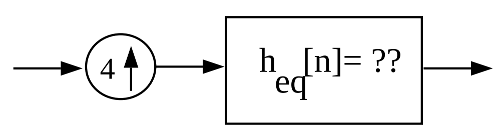

Figure 4(b).

Problem 2. [30 points]

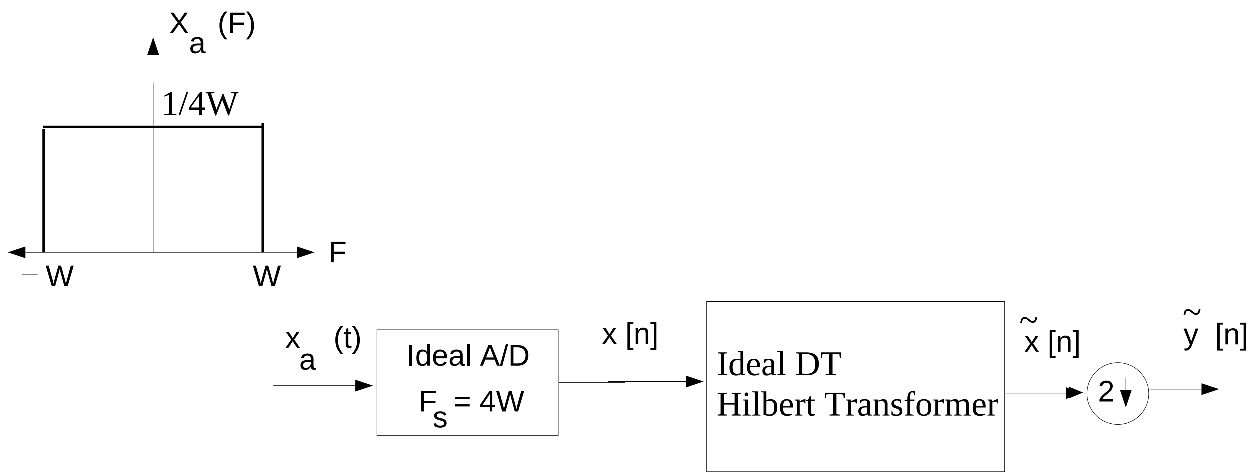

Figure 1.

The analog signal $x_a(t)$ with CTFT $X_a(F)$ shown above is input to the system above, where $x[n] = x_a(n/F_s)$ with $F_s = 4W$, and the Ideal Hibert Transformer is an LTI system with frequency response

$$H(\omega) = \begin{cases} j, & -\pi < \omega < 0 \\ -j, & 0 < \omega < \pi \end{cases}$$

(a) Plot the magnitude and phase of the DTFT of the output of the Hilbert Transformer $\tilde{x}[n]$, $\tilde{X}(\omega)$, over $-\pi < \omega < \pi$. That is, plot both $|\tilde{X}(\omega)|$ and $\angle\tilde{X}(\omega)$ (separate plots) over $-\pi < \omega < \pi$.

(b) Write a closed-form expression for $\tilde{x}[n]$. Recall that $\tilde{x}[n]$ is real-valued so it is required that your final answer be simplified to the point that it does not have a j in it.

(c) A complex-valued signal is created as $\hat{x}[n] = x[n] + j\tilde{x}[n]$. Plot the magnitude of the DTFT of $\hat{x}[n]$, $|\hat{X}(\omega)|$, over $-\pi < \omega < \pi$.

(d) For this part, a complex-valued signal is created from the output of the decimator as $\hat{y}[n] = \delta[n] + j\tilde{y}[n]$. Plot the magnitude of the DTFT of $\hat{y}[n]$, $|\hat{Y}(\omega)|$, over $-\pi < \omega < \pi$.

Problem 3. [40 points]

Let $x_{a1}(t)$, $x_{a2}(t)$, $x_{a3}(t)$, and $x_{a4}(t)$ be four real-valued (lowpass) signals having the same bandwidth, B, and with corresponding CTFT's $X_{a1}(F)$, $X_{a2}(F)$, $X_{a3}(F)$, and $X_{a4}(F)$ depicted in the block diagram on the next page, Figure 4. Each signal is sampled at the Nyquist rate of $F_s = 2B$. The three signals are processed and subsequently summed as shown in Figure 4. Defining $h_{LP}[n]$ as

$$h_{LP}(n) = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}, \qquad -\infty < n < \infty,$$

$\tilde{h}_{LP}[n]$ is the (ideal) Discrete-Time Hilbert Transform of $h_{LP}[n]$. (If necessary, see pages 657-659 of the text for both a frequency and time-domain description of the ideal DT Hilbert Transform.)

As indicated in Figure 4 on the next page, $h_1[n] = h_{LP}[n] + j\tilde{h}_{LP}[n]$. The respective impulse responses of each of the other three filters are $h_2[n] = e^{j\pi n}h_1[n]$, $h_3[n] = e^{j\frac{\pi}{2}n}h_1[n]$, $h_4[n] = e^{-j\frac{\pi}{2}n}h_1[n]$.

(a) Let $Y(\omega)$ denote the DTFT of the sum signal, $y[n]$, at the output. Plot the magnitude of $Y(\omega)$ over $-\pi < \omega < \pi$. Show as much detail as possible. *You do NOT need to show a lot of work in arriving at your answer. If you know what the system is doing, draw your answer and provide a brief explanation. Think about what's happening in the frequency domain – don't even think about doing any convolution.*

(b) Draw a block diagram of a system for recovering each of the four original sampled signals, $x_1[n]$, $x_2[n]$, $x_3[n]$, and $x_4[n]$, from the sum signal, $y[n]$. **In contrast to Exam 2, for this exam you have to provide a computationally efficient scheme for recovering each of the signals. You can ONLY use an interconnection of LTI filters to recover each signal. Specifically, you cannot use any of the following sub-systems:**

(i) You **cannot** use decimators. (Recall that filtering followed by decimation is inefficient since you throw away values of the computed filter output.)

(ii) You **cannot** take the real part of a signal. (That is also inefficient since that would imply that you computed the imaginary part of an output only to ultimately throw it away.)

(iii) You **cannot** use any kind of modulation, that is, multiplication a signal by a sinewave is **not** permitted.

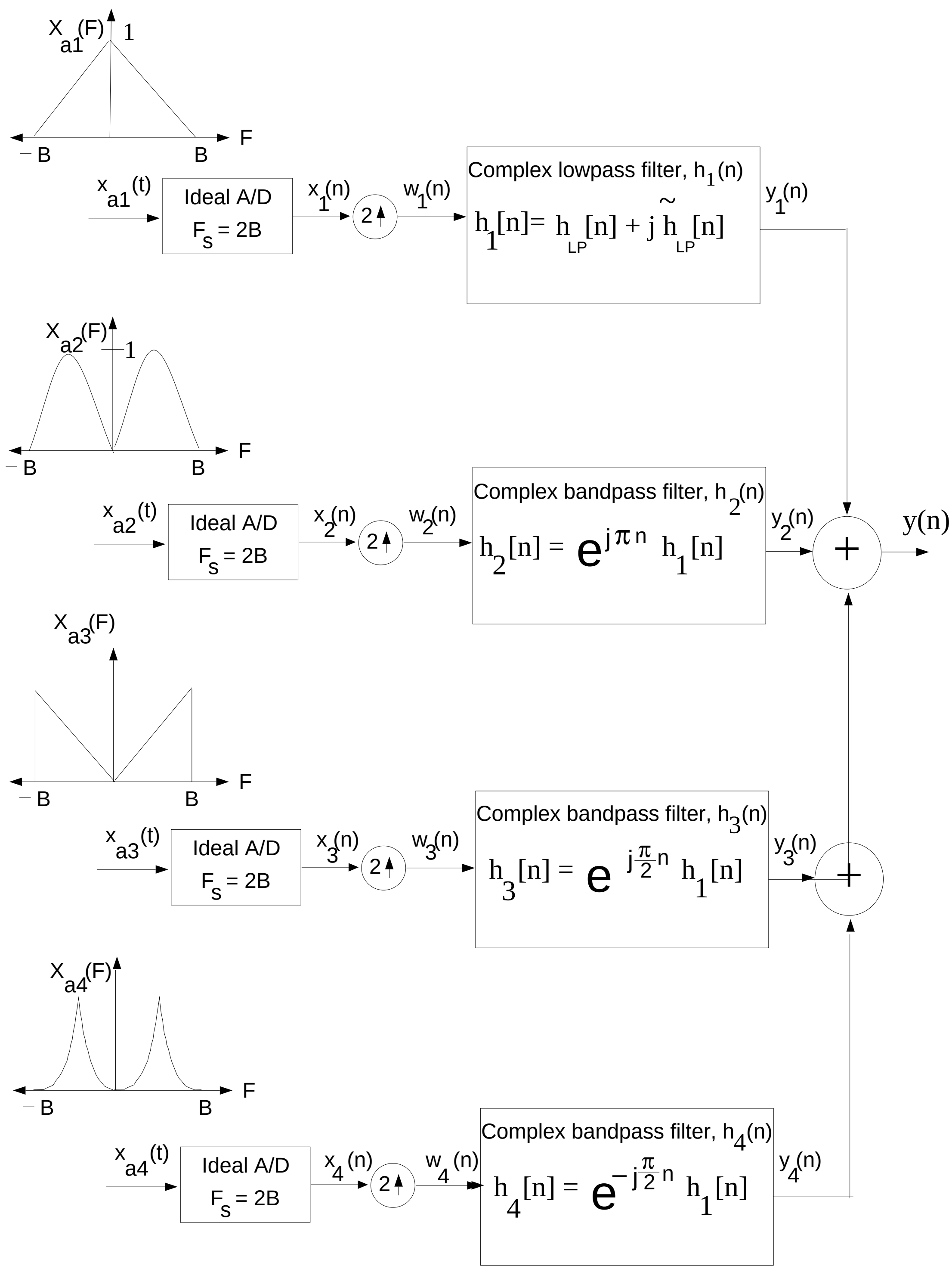

Figure 4: Digital subbanding of four real-valued signals each sampled at Nyquist rate.