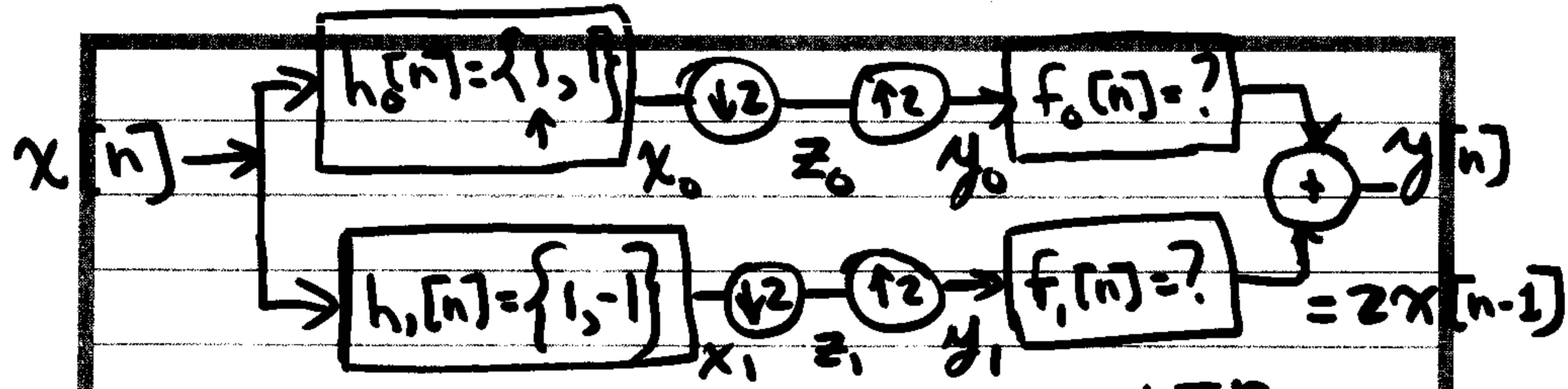

Note: $h_1[n] = (-1)^n h_0[n] = e^{j\pi n} h_0[n]$

$$\Rightarrow H_1(\omega) = H_0(\omega - \pi)$$

• Work in freq. domain:

$$X_i(\omega) = H_i(\omega) X(\omega) \qquad i = 0, 1$$

$$Z_i(\omega) = \frac{1}{2}\left\{X_i\left(\frac{\omega}{2}\right) + X_i\left(\frac{\omega-2\pi}{2}\right)\right\}$$

$$\boxed{Z_i(\omega) = \frac{1}{2}\left\{H_i\left(\frac{\omega}{2}\right)X\left(\frac{\omega}{2}\right) + H_i\left(\frac{\omega-2\pi}{2}\right)X\left(\frac{\omega-2\pi}{2}\right)\right\}}$$

$$Y_i(\omega) = Z_i(2\omega) \qquad i = 0, 1$$

$$Y_i(\omega) = \frac{1}{2}\left\{H_i(\omega)\, X(\omega) + H_i(\omega-\pi)\, X(\omega-\pi)\right\}$$

$$Y(\omega) = F_0(\omega)\, Y_0(\omega) + F_1(\omega)\, Y_1(\omega)$$

$$= \frac{1}{2}\left\{H_0(\omega) F_0(\omega) + H_1(\omega)\, F_1(\omega)\right\} X(\omega)$$

$$+ \frac{1}{2}\left\{H_0(\omega-\pi)\, F_0(\omega) + H_1(\omega-\pi)\, F_1(\omega)\right\} X(\omega-\pi)$$

Substitute $H_1(\omega) = H_0(\omega-\pi)$

$$\Rightarrow H_1(\omega-\pi) = H_0(\omega-2\pi) = H_0(\omega)$$

Desire: $Y(\omega) = 2\, e^{-j\omega}\, X(\omega)$

$(y[n] = 2x[n-1])$

(A) $F_0(\omega) H_0(\omega) + F_1(\omega) H_0(\omega-\pi) = 4e^{-j\omega}$

(B) $F_0(\omega) H_0(\omega-\pi) + F_1(\omega) H_0(\omega) = 0$

(A) $a x + b y = d$ $\quad$ b (A) $- a$ (B)

(B) $b x + a y = 0$

a (A) $- b$ (B) $= (a^2 - b^2) x = a d$

$x = \dfrac{a d}{a^2 - b^2}$ $\Rightarrow y = \dfrac{bd}{b^2 - a^2}$

$$F_0(\omega) = \frac{H_0(\omega)\, 4e^{-j\omega}}{H_0^2(\omega) - H_0^2(\omega-\pi)}$$

$$\{\underset{\uparrow}{1},2,1\} - \{\underset{\uparrow}{1},-2,1\} \longleftrightarrow H_0^2(\omega) - H_0^2(\omega-\pi)$$

$$h_0[n] = \{\underset{\uparrow}{1},1\}$$

$$= \{\underset{\uparrow}{0},4,0\}$$

$$h_1[n] = \{1,-1\}$$

$$= 4\delta[n-1] \longleftrightarrow 4e^{-j\omega}$$

Substituting: $F_0(\omega) = H_0(\omega)$

$$f_0(n) = \{\underset{\uparrow}{1},1\}$$

$$F_1(\omega) = -H_0(\omega-\pi) = -H_1(\omega)$$

$$f_1[n] = \{-1,1\} = \{\underset{\uparrow}{-1},1\}$$

$$= -h_1[n] =$$