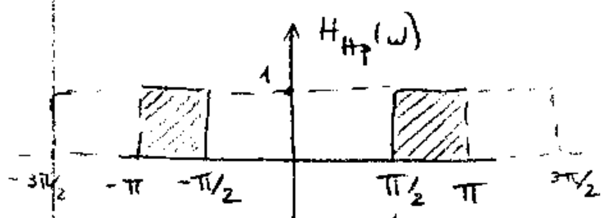
flip using Noble's first identity

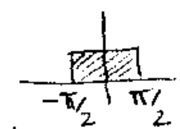
P1) a) $\rightarrow (2\uparrow) \rightarrow h_{HP}(n) \rightarrow (2\uparrow) \rightarrow h_{HP}(n)$

$\Rightarrow \quad \rightarrow (2\uparrow) \rightarrow (2\uparrow) \rightarrow \underbrace{\tilde{h}_{HP}(n) \rightarrow h_{HP}(n)}_{h(n)} \rightarrow$

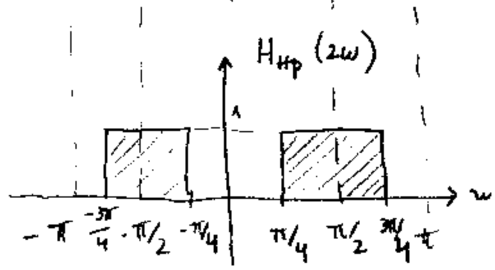
where $\tilde{H}_{HP}(\omega) = H_{HP}(2\omega)$

$\therefore \quad H(\omega) = H_{HP}(2\omega)\, H_{HP}(\omega).$



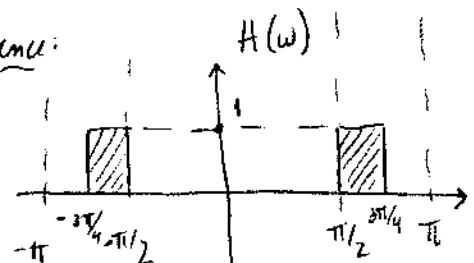


shifted by π to the right (since $(-1)^n = e^{j\pi n}$)

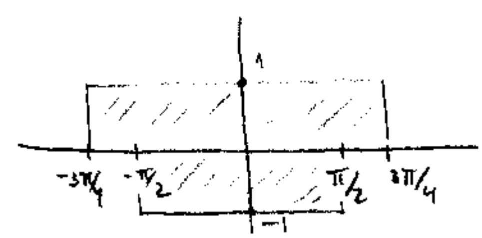


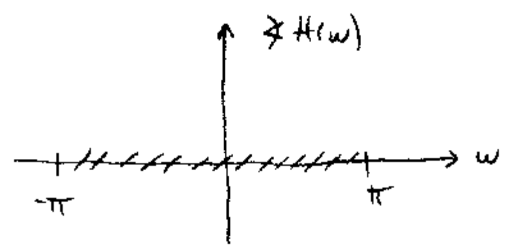
(all periodic 2π)

Hence:



$\equiv$





Hence:

$$h(n) = \frac{\sin\left(\frac{3\pi}{4}n\right)}{\pi n} - \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$

b) $\longrightarrow (2\uparrow) \longrightarrow h_1(n) \longrightarrow (2\uparrow) \longrightarrow h_2(n)$

$\therefore \quad \longrightarrow (2\uparrow) \longrightarrow (2\uparrow) \longrightarrow \underbrace{\tilde{h}_1(n) \longrightarrow h_2(n)}_{h_{eq}(n)} \longrightarrow$

$$h_{eq}(n) = \tilde{h}_1(n) * h_2(n) \qquad \left(\tilde{H}_1(\omega) = H_1(2\omega) \right)$$

$$= \{ \underset{\uparrow}{1}, 0, -1, 0, 1, 0, -1 \} * \{ \underset{\uparrow}{1}, -1 \}$$

$$= \{ \underset{\uparrow}{1}, -1, -1, 1, 1, -1, -1, 1 \}$$

P2)
$$x_1(n) \longrightarrow (2\downarrow) \xrightarrow{z_1(n)} (2\uparrow) \longrightarrow y_1(n)$$

$$z_1(n) = x_1(2n) \qquad y_1(n) = \begin{cases} z_1(n/2) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$\therefore \quad y_1(n) = \begin{cases} x_1(n) & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \qquad (L \text{ but not } TI)$$

Now: $h_0(n) = \{\underset{\uparrow}{1}, 1\} \Rightarrow x_1(n) = x(n) + x(n-1)$

Then:
$$y_1(n) = \begin{cases} x(n) + x(n-1) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_1(n) \longrightarrow \boxed{f_0(n)} \longrightarrow \tilde{y}_1(n) \quad ; \quad f_0(n) = \{\underset{\uparrow}{1}, 1\} \Rightarrow \tilde{y}_1(n) = y_1(n) + y_1(n-1)$$

$$\therefore \quad \tilde{y}_1(n) = \begin{cases} x(n) + x(n-1) & n \text{ even} \\ x(n-1) + x(n-2) & n \text{ odd} \end{cases}$$

In the same way:

$h_1(n) = \{\underset{\uparrow}{1}, -1\} \Rightarrow x_2(n) = x(n) - x(n-1)$

Then:
$$y_2(n) = \begin{cases} x(n) - x(n-1) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_2(n) \longrightarrow \boxed{f_1(n)} \longrightarrow \tilde{y}_2(n) \quad ; \quad f_1(n) = \{\underset{\uparrow}{-1}, 1\} \Rightarrow \tilde{y}_2(n) = -y_2(n) + y_2(n-1)$$

$$\therefore \quad \tilde{y}_2(n) = \begin{cases} -x(n) + x(n-1) & n \text{ even} \\ x(n-1) - x(n-2) & n \text{ odd} \end{cases}$$

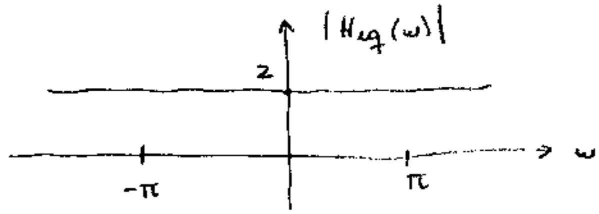
Hence:
$$\underline{y(n) = \tilde{y}_1(n) + \tilde{y}_2(n) = 2\,x(n-1)}$$

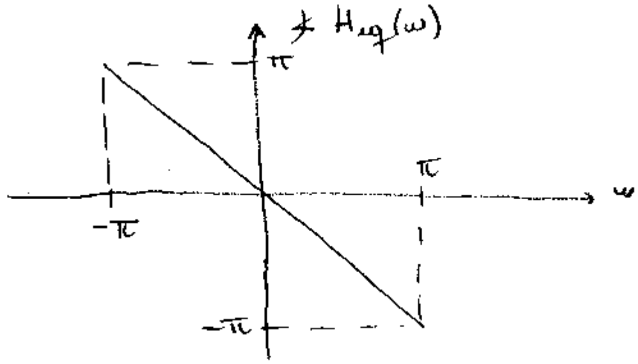
which can be represente as an LTI system with:

$$y(n) = 2x(n-1) \longrightarrow \underline{h_{eq}(n) = 2\delta(n-1)}$$

Hence: $$H_{eq}(\omega) = 2e^{-j\omega}$$

ie





73) $$H_a(s) = \frac{\Omega_c}{s+\Omega_c} \quad ; \quad \Omega_c = 1$$

$$s = \frac{z-j}{z+j}$$

a) $$H(z) = \left.\frac{1}{s+1}\right|_{s=\frac{z-j}{z+j}} = \frac{z+j}{2z}$$

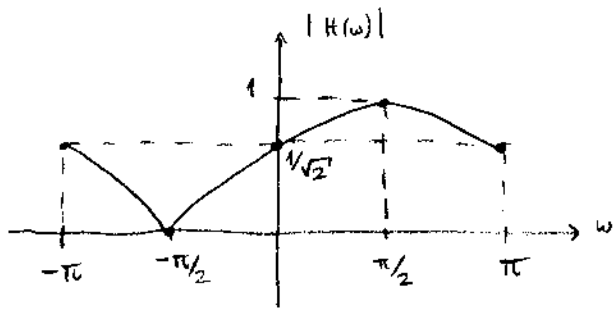
$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = \frac{e^{j\omega}+j}{2e^{j\omega}} = \frac{1}{2}\left(1+e^{j(\pi/2-\omega)}\right)$$
$$= \frac{1}{2}\left((1+\operatorname{sen}\omega)+j\cos\omega\right)$$

Hence:

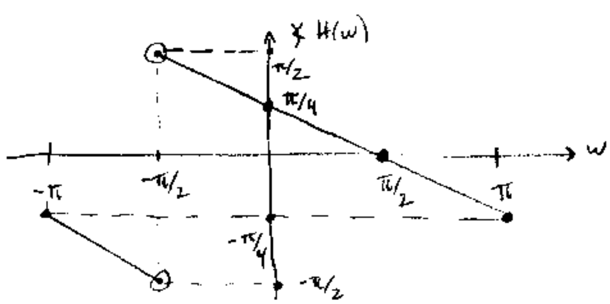
$$|H(\omega)| = \frac{1}{2}\sqrt{\cos^2\omega + (\operatorname{sen}\omega+1)^2} = \sqrt{\frac{1+\operatorname{sen}\omega}{2}}$$

$$\measuredangle H(\omega) = tg^{-1}\left(\frac{\cos\omega}{1+\operatorname{sen}\omega}\right)$$

ie:



(not symmetrical since $h(n)$ is not real)



$$\left(\lim_{\omega \uparrow -\pi/2} \frac{\cos\omega}{1+\operatorname{sen}\omega} \longrightarrow -\infty\right.$$

$$\left.\lim_{\omega \downarrow -\pi/2} \frac{\cos\omega}{1+\operatorname{sen}\omega} \longrightarrow +\infty\right)$$

b) pole at $z = 0$ inside $\odot_1$

$\therefore$ <u>stable</u>

c) $$H(z) = \frac{Y(z)}{X(z)} = \frac{z+j}{2z} = \frac{1 + j z^{-1}}{2}$$

$$\therefore \quad 2\,Y(z) = X(z)\left\{ 1 + j z^{-1} \right\}$$

$$\Rightarrow \quad y(n) = \frac{1}{2}\left(x(n) + j\, x(n-1) \right)$$