EE538 Digital Signal Processing I — Session 29
Exam 2 — Live: Mon., Oct. 28, 2002

Cover Sheet

Test Duration: 50 minutes.
Open Book but Closed Notes.
Calculators allowed but NOT needed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

No.	Topic(s) of Problem	Points
1.	Multi-Stage Interpolation	30
2.	Principles of Upsampling & Downsampling (esp. Frequency Domain Analysis)	40
3.	IIR Filter Design Via Bilinear Transform Method	30

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GIVEN NOBLE's IDENTITIES TO USE IN PROBLEM 1.

(a) If $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) satsifies $E(\omega) = G(M\omega)$, the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

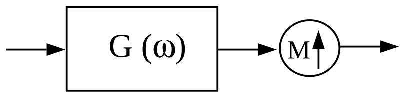

Figure 1(a).

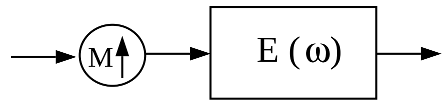

Figure 1(b).

(b) If $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) satisfies $F(\omega) = H(M\omega)$, the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity.

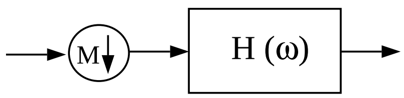

Figure 2(a).

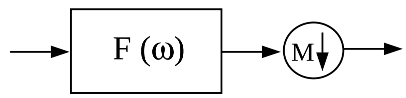

Figure 2(b).

Problem 1. [30 points]

(a) Determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). Plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$. *Hint:* Analyze the system of Figure 3(a) in the frequency domain using Noble's First Identity.

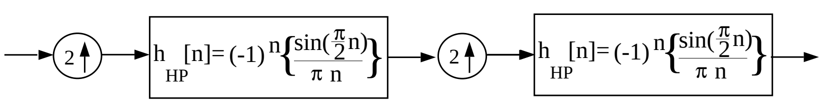

Figure 3(a).

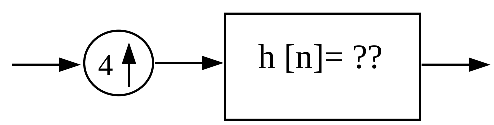

Figure 3(b).

(b) Determine the numerical values of the impulse response $h_{\text{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). *Hint:* Analyze the system of Figure 4(a) in the time domain using Noble's First Identity.

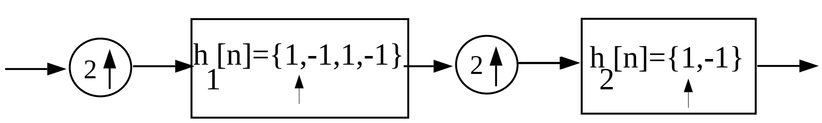

Figure 4(a).

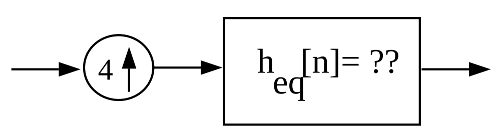

Figure 4(b).

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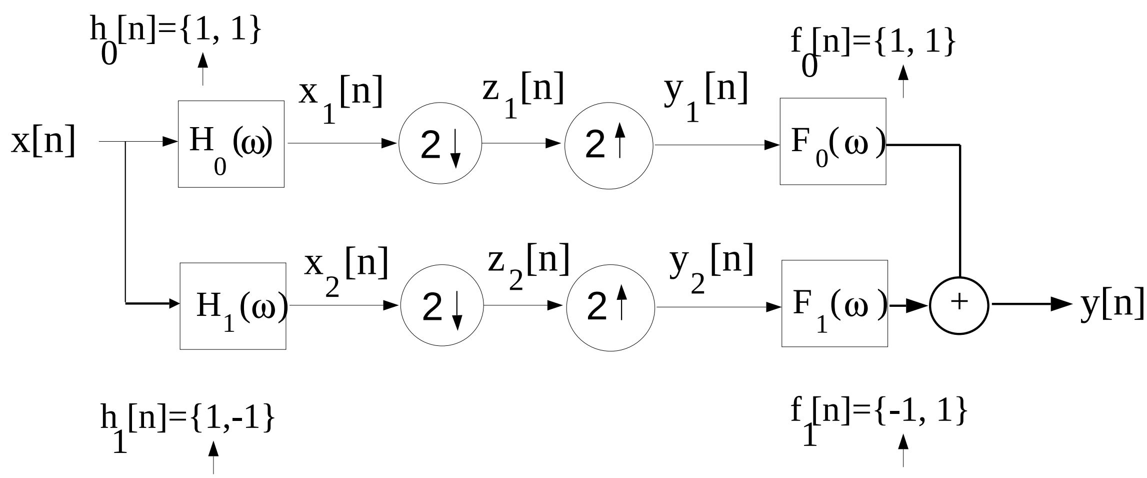
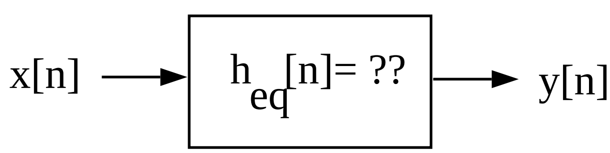
Problem 2. [40 points]

In the system below, each of the analysis filters, $h_0[n]$ and $h_1[n]$, and each of the two synthesis filters, $f_0[n]$ and $f_1[n]$, is a causal FIR filter of length 2 with the specific values indicated. The arrow denotes the value at $n = 0$. (See the hints at the bottom of the page.)



where $z_i[n] = x_i[2n], \;\; i = 1, 2$ and $y_i[n] = \sum_{\ell=-\infty}^{\infty} z_i[\ell]\delta[n - 2\ell], \;\; i = 1, 2.$

You are *given* that the overall system from the input $x[n]$ to the output $y[n]$ is an LTI (linear and time-invariant) system. Thus, the overall system can be reduced to the equivalent system below



(a) Determine the impulse response, $h_{eq}[n]$, of the equivalent system.

(b) Plot the magnitude of the frequency response of the overall system, $|H_{eq}(\omega)|$, over $-\pi < \omega < \pi$ showing as much detail as possible.

(c) Plot the phase of the frequency response of the overall system, $\angle H_{eq}(\omega)$, over $-\pi < \omega < \pi$ showing as much detail as possible.

Hint 1. The series combination of a downsampler followed by an upsampler does NOT reduce to an identity transformation – they don't "cancel" each other.

Hint 2. This problem can be solved either in the time domain or the frequency domain with about equal complexity. Note $f_0[n] = h_0[n]$ and $f_1[n] = -h_1[n]$ and $h_1[n] = (-1)^n h_0[n]$.

Hint 3. The final answer for $h_{eq}[n]$ reduces to a simple expression.

Problem 3. [30 points]

An analog Butterworth filter of order $N = 1$ with a 3-dB cut-off at $\Omega_c = 1$ has the following transfer function (Laplace Transform):

$$H_a(s) = \frac{1}{s+1} \tag{1}$$

A digital filter is synthesized via the following bilinear transformation *which is different from the one used in class or the textbook.*

$$s = \frac{z-j}{z+j} \tag{2}$$

(a) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform in (2) to the analog filter described by (1). Plot the magnitude of the frequency response of the resulting digitial filter over $-\pi < \omega < \pi$ showing as much detail as possible.

(b) Is the resulting digital filter stable?

(c) Determine the difference equation for implementing the resulting digital lowpass filter.