EE538 **Exam 2** **Fall 2001**
Digital Signal Processing I **30 October 2001**

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators **not** allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

No.	**Topic(s) of Problem**	**Points**
1.	Principles of Upsampling & Downsampling (esp. Frequency Domain Analysis)	30
2.	Multi-Stage Interpolation	30
3.	IIR Filter Design Via Bilinear Transform Method	40

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Problem 1. [30 points]

(a) Find $E(\omega)$ in Figure 1(b) in terms of $G(\omega)$ in Figure 1(a) such that the I/O relationship of the system in Figure 1(b) is exactly the same as the I/O relationship of the system in Figure 1(a). This result is known as Noble's First Identity.

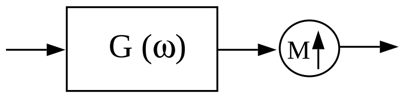

Figure 1(a).

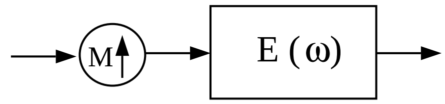

Figure 1(b).

(b) Find $F(\omega)$ in Figure 2(b) in terms of $H(\omega)$ in Figure 2(a) such that the I/O relationship of the system in Figure 2(b) is exactly the same as the I/O relationship of the system in Figure 2(a). This result is known as Noble's Second Identity. *Show all work.*

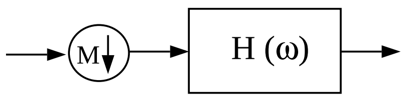

Figure 2(a).

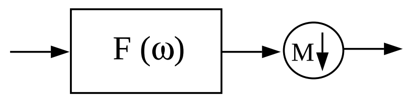

Figure 2(b).

Problem 2. [30 points]

(a) Using the result from Part (a) of Problem 1, determine the impulse response $h[n]$ in Figure 3(b) so that the I/O relationship of the system in Figure 3(b) is exactly the same as the I/O relationship of the system in Figure 3(a). You must show all work and logic in arriving at your answer to receive full credit. **In addition**, plot the magnitude AND the phase (two separate plots) of the DTFT of $h[n]$ over $-\pi < \omega < \pi$.

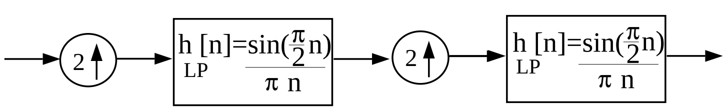

Figure 3(a).

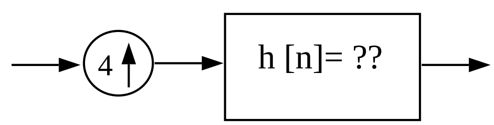

Figure 3(b).

(b) Once again using the result from part (a) Problem 1, determine the numerical values of the impulse response $h_{\text{eq}}[n]$ in Figure 4(b) so that the I/O relationship of the system in Figure 4(b) is exactly the same as the I/O relationship of the system in Figure 4(a). You must show all work and logic in arriving at your answer to receive full credit. **In addition**, plot the magnitude AND the phase (two separate plots) of the DTFT of $h_{\text{eq}}[n]$ over $-\pi < \omega < \pi$.

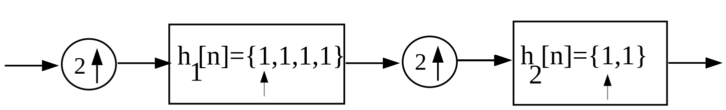

Figure 4(a).

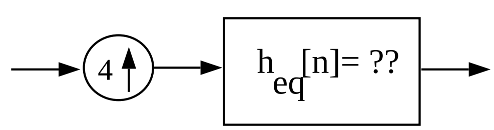

Figure 4(b).

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Problem 3. [40 points]

A second-order digital filter is to be designed from an analog filter having two poles in the s-plane at $-1+2j$ and $-1-2j$ and two zeros at j and $-j$, via the bilinear transformation method characterized by the mapping

$$s = \frac{z-1}{z+1}$$

(a) Is the resulting digital filter (BIBO) stable? Briefly explain why or why not.

(b) Denote the frequency response of the resulting digital filter as $H(\omega)$ (the DTFT of its impulse response). You are given that in the range $0 < \omega < \pi$, there is only one value of ω for which $H(\omega) = 0$. Determine that value of ω.

(c) Draw a pole-zero diagram for the resulting **digital** filter. Give the exact locations of the poles and zeros of the digital filter in the z-plane.

(d) Plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. You are given that $H(0) = 0.8$. Be sure to indicate any frequency for which $|H(\omega)| = 0$. Also, specifically note the numerical value of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

(e) Determine the difference equation for the resulting digital filter.