- **Note on Soln to Problem 1 on Exam 1 from Fall 2015**

$$h[n] = \frac{1}{p}\left\{1 + (p^2-1)\, p^n u[n]\right\}$$

p is real-valued with $-1 < p < 1$

- Take Z-Transform

$$H(z) = \frac{1}{p}\left\{\underbrace{\frac{z-p}{z-p}}_{1} + (p^2-1)\frac{z}{z-p}\right\}$$

$$= \frac{1}{p}\left\{\frac{z - p + p^2 z - z}{z-p}\right\} = \frac{-1 + pz}{z-p}$$

$$= p\,\frac{(z - 1/p)}{z-p} = p\,\frac{\left(z - \frac{1}{p}\right)}{z-p} \qquad \text{all-pass}$$

- Thus, for $p = 1/2$: $H_1(z) = \frac{1}{2} \frac{(z-2)}{z - \frac{1}{2}}$

  - for $p = -\frac{1}{2}$: $H_2(z) = -\frac{1}{2} \frac{(z+2)}{z + \frac{1}{2}}$

- And:

$$H(z) = H_1(z)\, H_2(z)$$

$$= -\frac{1}{4} \left\{ \frac{z^2 - 4}{z^2 - \frac{1}{4}} \right\}$$

$$H(\omega)\Big|_{\omega=0} = H(z)\Big|_{z=1} = -\frac{1}{4} \left\{ \frac{1-4}{1-\frac{1}{4}} \right\} = \frac{-3}{-3} = 1$$

$$H(\omega)\Big|_{\omega=\pi/2} = H(z)\Big|_{z=j} = -\frac{1}{4} \left\{ \frac{-1-4}{-1-\frac{1}{4}} \right\} = \frac{-5}{5} = -1$$

- Since $h[n]$ is real-valued $H(-\omega) = H^*(\omega)$

- thus $H(\omega)\Big|_{\omega=-\pi/2} = -1$

Finally,

$$H(\omega)\Big|_{\omega=\pi} = H(z)\Big|_{z=-1} = -\frac{1}{4}\left\{\frac{1-4}{1-\frac{1}{4}}\right\} = 1$$

- Thus:

$$y[n] = 4\,H(0) + 3\,H\left(\frac{\pi}{2}\right)e^{j\frac{\pi}{2}n} + 2\,H\left(-\frac{\pi}{2}\right)e^{-j\frac{\pi}{2}n} + H(\pi)e^{j\pi n}$$

$$= 4 + 3(-1)e^{j\frac{\pi}{2}n} + 2(-1)e^{-j\frac{\pi}{2}n} + e^{j\pi n}$$

$$= 4 - 3(j)^n - 2(-j)^n + (-1)^n$$