<u>Correlation</u> <u>Review</u> (1 pg)

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$r_{xx}[m] =$ $\qquad r_{hh}[m] \qquad r_{yy}[m]$

$x[m] * x^*[-m]$

$$r_{yx}[m] = h[m] * r_{xx}[m]$$

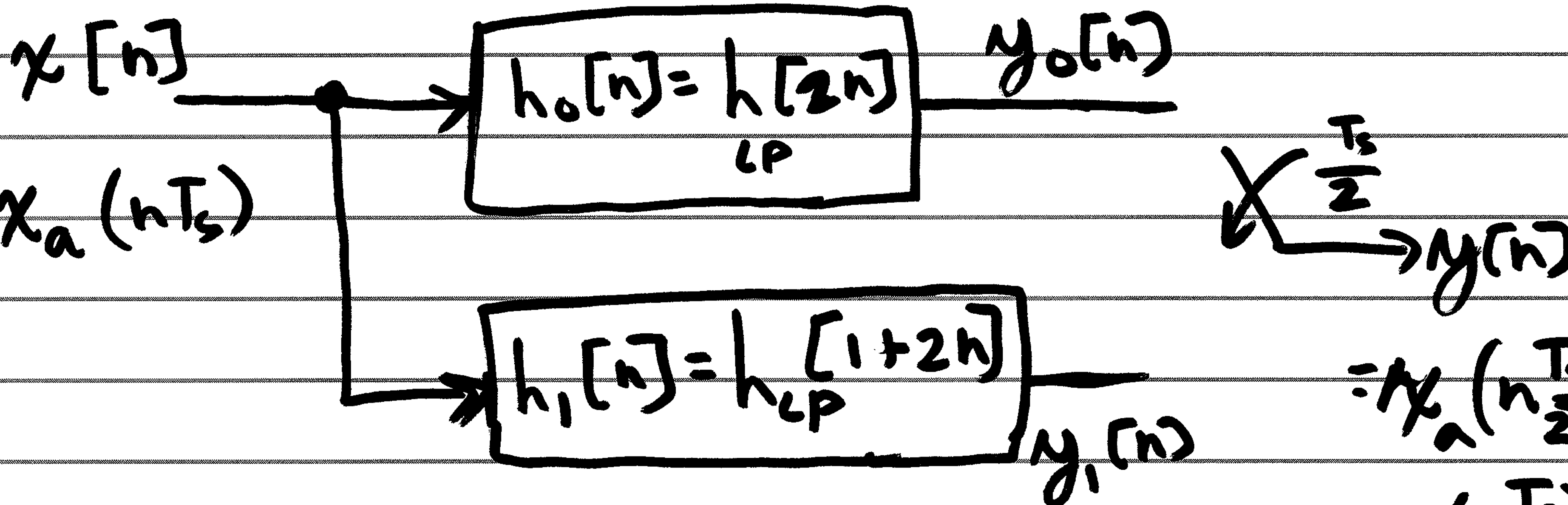
$$r_{yy}[m] = r_{hh}[m] * r_{xx}[m]$$

$$r_{xx}[m] \overset{DTFT}{\longleftrightarrow} S_{xx}(\omega) = |X(\omega)|^2$$

$$S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$$

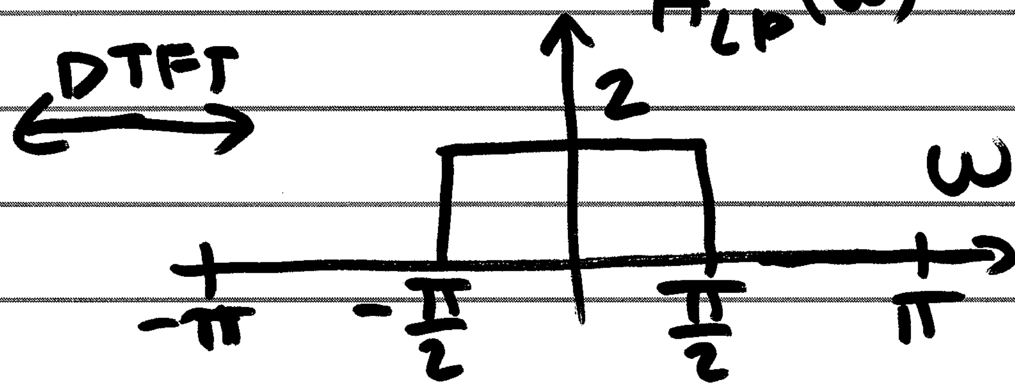
Efficient Upsampling Review

$$H_0(\omega) = \frac{1}{2} H_{LP}\left(\frac{\omega}{2}\right) + \frac{1}{2} H_{LP}\left(\frac{\omega - 2\pi}{2}\right)$$



$= x_a\left(n\frac{T_s}{2}\right)$

$h_{LP}[n]$ ⇒ ideal case $H_{LP}(\omega)$



In ideal case: $H_0(\omega) = 1$ for all ω
⇒ makes sense!

③

$$h_1[n] = h_{LP}[1+2n] \overset{DTFT}{\longleftrightarrow} H_1(\omega) = ?$$

$$H_1(\omega) = \sum_{n=-\infty}^{\infty} h_{LP}[1+2n]\, e^{-j\omega n}$$

$$= \sum_{\substack{n' \\ n' \text{ odd}}} h_{LP}[n']\, e^{-j\omega\left(\frac{n'-1}{2}\right)}$$

$$\boxed{\begin{aligned} n' &= 2n+1 \\ n &= \frac{n'-1}{2} \end{aligned}}$$

$$= \sum_{n'} \frac{1}{2}\left(1-(-1)^{n'}\right) h_{LP}[n']\, e^{-j\omega\left(\frac{n'-1}{2}\right)}$$

$= \Big\{ \frac{1}{2} \sum_n h_{LP}[n]\, e^{-jn\frac{\omega}{2}}$ [redacted] $\frac{1}{2} \sum_n h_{LP}[n]\, e^{-jn\frac{(\omega-2\pi)}{2}} \Big\} \cdot e^{+j\frac{\omega}{2}}$

$$h_1[n] = h_{LP}[1+2n] \overset{DTFT}{\longleftrightarrow}$$

$$H_1(\omega) = \left\{ \frac{1}{2} H_{LP}\left(\frac{\omega}{2}\right) - \frac{1}{2} H_{LP}\left(\frac{\omega - 2\pi}{2}\right) \right\} \cdot e^{+j\frac{\omega}{2}}$$

$e^{+j\frac{\omega}{2}}$ corresponds to half-sample offset back in continuous time $\Rightarrow$ makes sense!

Equation to DT System Review

$$x(t) = \sum_{k=-\infty}^{\infty} b[k]\, p(t - kT_0)$$

$$x[n] = x(nT_s) \quad \text{where: } T_s = \frac{T_0}{2}$$

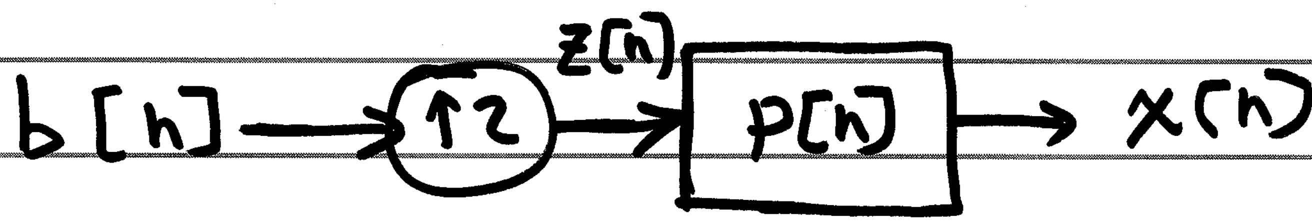
$$x[n] = x\left(n\frac{T_0}{2}\right)$$

$$= \sum_{k=-\infty}^{\infty} b[k]\, p\left(n\frac{T_0}{2} - 2k\frac{T_0}{2}\right)$$

$$p[n] = p\left(n\frac{T_0}{2}\right)$$

$$x[n] = \sum_{k} b[k]\, p[n-2k]$$

$$x[n] = \sum_k b[k]\, p[n-2k]$$



$$z[n] = \sum_k b[k]\, \delta[n-2k]$$

$$= \{\ldots, b[-1], 0, \underset{\uparrow}{b[0]}, 0, b[1], 0, \ldots\}$$

Note: $Z(\omega) = B(2\omega)$

$$x_0[n] = x[2n] = \sum_k b[k]\, p[\underbrace{2n-2k}_{2(n-k)}]$$

$$p_0[n] = p[2n]$$

$$x_0[n] = b[n] * p_0[n]$$

$$x_1[n] = x[2n+1] = \sum_k b[k]\, p[\underbrace{2n+1-2k}_{1+2(n-k)}]$$

$$p_1[n] = p[1+2n]$$

$$x_1[n] = b[n] * p_1[n]$$