ECE 538 Exam 1 Solns SF 2005

Soln to Prob. 1 (1)

$\underline{b_1[0]}$:

$$-4, -2, 2, 4$$
$$\odot \; 1, -1, 1, -1$$
$$\frac{-4+2+2-4}{4} = -1 \Rightarrow \{0,1\}$$

$\underline{b_1[1]}$:

$$4, -2, 2, -4$$
$$\odot \; 1, -1, 1, -1$$
$$\frac{4+2+2+4}{4} = 3 \Rightarrow \{1,1\}$$

$\underline{b_2[0]}$:

$$-4, -2, 2, 4$$
$$\odot \; 1, 1, -1, -1$$
$$\frac{-4-2-2-4}{4} = -3 \Rightarrow \{0,0\}$$

$\underline{b_2[1]}$:

$$4, -2, 2, -4$$
$$\odot \; 1, 1, -1, -1$$
$$\frac{4-2-2+4}{4} = 1 \Rightarrow \{1,0\}$$

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Prob. 2 Soln

$$x_r\left(n\frac{T_s}{L}\right) = \sum_k x[k]\, g(t - kT_s)\Big|_{t = n\frac{T_s}{L}}$$

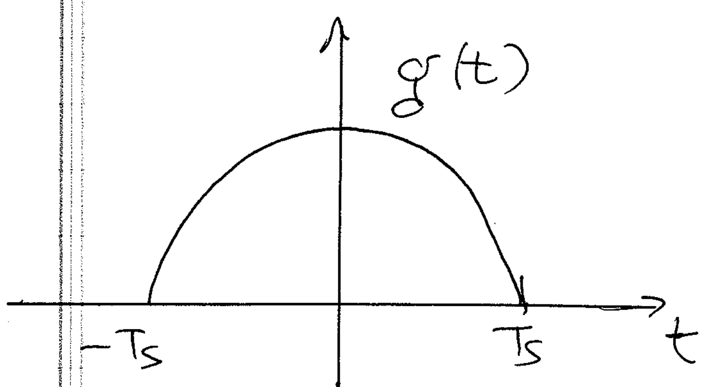
$$= \sum_k x[k]\, g\left((n-kL)\frac{T_s}{L}\right)$$

$$= \sum_k x[k]\, g[n-kL]$$

where: $g[n] = g\left(n\frac{T_s}{L}\right)$

$$x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{g[n]} \rightarrow x_r[n]$$

Thus: $h[n] = g[n] = g\left(n\frac{T_s}{L}\right)$



(a) $L = 1$

$$h[n] = \delta[n]$$

(b) $L = 2$:

$$h[n] = \cos\left(\frac{\pi}{2T_s}\,\frac{nT_s}{2}\right)$$

$$= \cos\left(\frac{\pi}{4}n\right)\left\{u[n+2] - u[n-2]\right\}$$

$$= \left\{\frac{1}{\sqrt{2}}, \underset{\uparrow}{1}, \frac{1}{\sqrt{2}}\right\}$$

(c) $L = 4$:

$$h[n] = \cos\left(\frac{\pi}{2T_s}\,\frac{nT_s}{4}\right)$$

$$= \cos\left(\frac{\pi}{8}n\right)\left\{u[n+4] - u[n-4]\right\}$$

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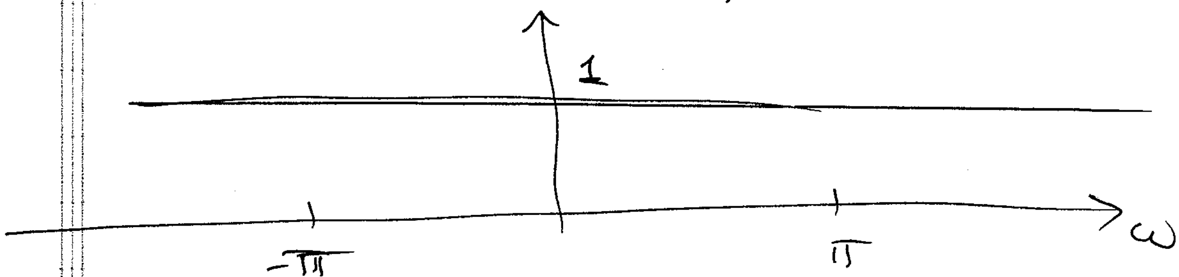
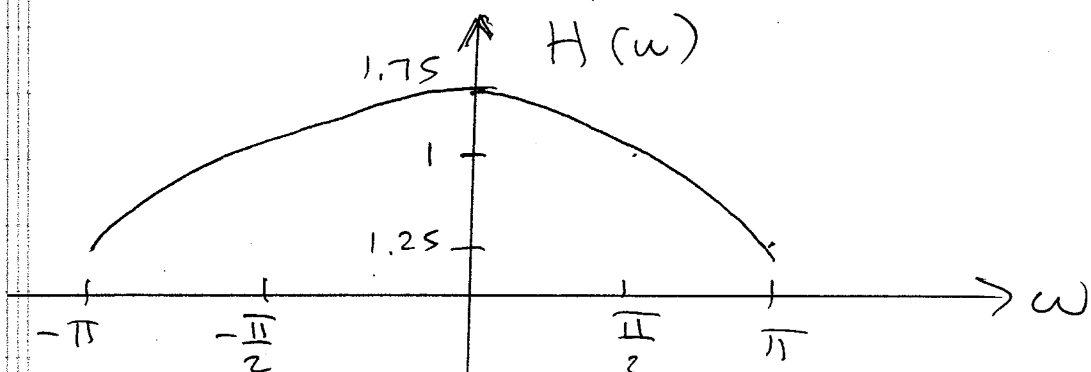
(a) $L = 1$: $\quad H(\omega) = 1 \quad \forall \omega$



(b) $H(\omega) = 1 + \sqrt{2}\cos(\omega)$

$\approx 1 + .7\cos(\omega)$

$\approx 1 + \frac{3}{4}\cos(\omega)$

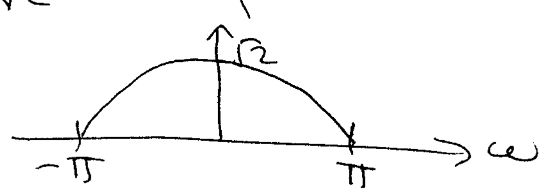


(i) $h_0[n] = h[2n] = \delta[n]$ $\Big\}$ see answer to (a) for plot

(ii) $h_1[n] = h[2n+1] = \left\{\frac{1}{\sqrt{2}}, \underset{\uparrow}{\frac{1}{\sqrt{2}}}\right\}$

$$H_1(\omega) = \frac{1}{\sqrt{2}}\left\{e^{j\omega} + 1\right\} = \frac{1}{\sqrt{2}} e^{j\frac{\omega}{2}}\left\{e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right\}$$

$$H_1(\omega) = \sqrt{2}\cos\left(\frac{\omega}{2}\right) e^{j\frac{\omega}{2}}$$



Prob. 2 (cont.)

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(c) $h[n] = \cos\left(\frac{\pi}{8} n\right)\left\{u[n+4] - u[n-4]\right\}$

- use modulation property:
- first, from Table:

$$u[n+4] - u[n-4] \overset{DTFT}{\longleftrightarrow} \frac{\sin\left(\frac{9}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$$

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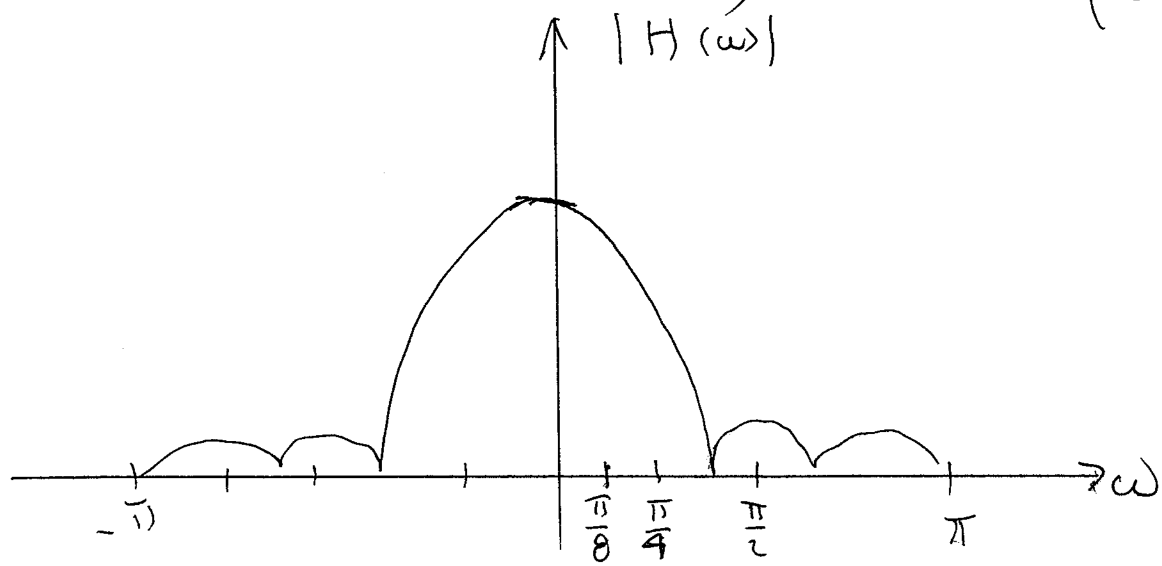
Thus:

$$H(\omega) = .5\,\frac{\sin\left(\frac{9}{2}\left(\omega - \frac{\pi}{8}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{\pi}{8}\right)\right)} + .5\,\frac{\sin\left(\frac{9}{2}\left(\omega + \frac{\pi}{8}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{\pi}{8}\right)\right)}$$



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Sol'n to Prob. 3

$$y(t) = x(t) * g(t)$$

$$= \left\{ \sum_{k=-\infty}^{\infty} b[k] \; p(t) * \delta(t - kT_0) \right\} * g(t)$$

$$= \left\{ \sum_{k=-\infty}^{\infty} b[k] \, \delta(t - kT_0) \right\} * \big(p(t) * g(t) \big)$$

$$= \sum_k b[k] \, \delta(t - kT_0) * f(t)$$

$$f(t) = p(t) * g(t)$$

$$= \sum_k b[k] \, f(t - kT_0)$$

$$y(nT_0) = \sum_k b[k] \, f[n-k]$$

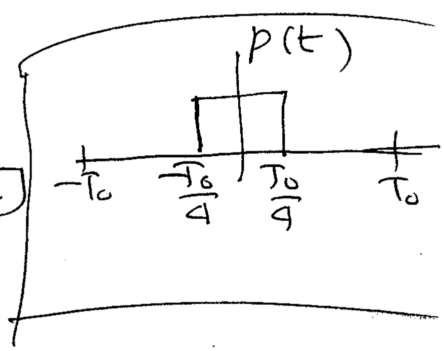
where: $f[n] = f(nT_0)$

$$f(t) = p(t) * \big(\delta(t) + e^{j\theta} \delta(t - T_0) \big)$$

$$= p(t) + e^{j\theta} p(t - T_0)$$

$$f[n] = p[n] + e^{j\theta} p[n-1]$$

where: $p[n] = p(nT_0) = \delta[n]$



Sol'n to Prob. 3 (cont.)

$$h[n] = \delta[n] + e^{j\theta}\,\delta[n-1]$$

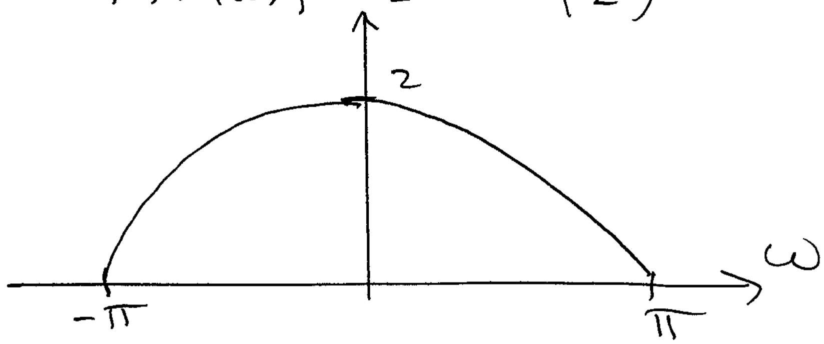
(i) $\theta = 0$ $\quad h[n] = \{\underset{\uparrow}{1}, 1\}$

$$H(\omega) = 1 + e^{j\theta} e^{-j\omega}$$

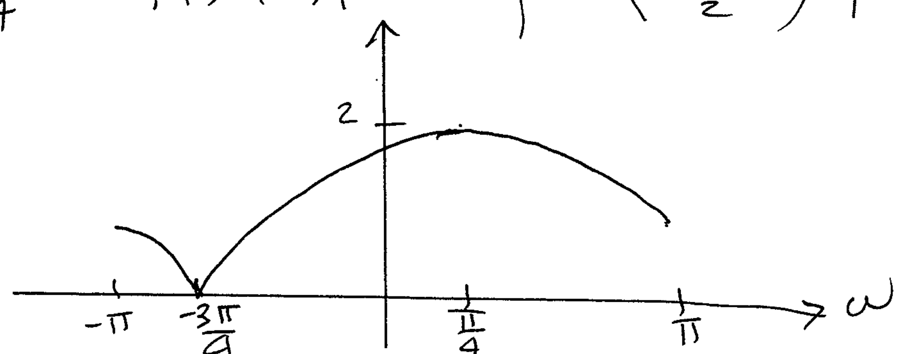
$$= 2 e^{j\frac{(\theta-\omega)}{2}} \cos\left(\frac{\theta-\omega}{2}\right)$$

$$|H(\omega)| = 2\left|\cos\left(\frac{\omega-\theta}{2}\right)\right|$$

(i) $\theta = 0$: $\quad |H(\omega)| = 2\cos\left(\frac{\omega}{2}\right)$



(ii) $\theta = \frac{\pi}{4}$ $\quad |H(\omega)| = 2\left|\cos\left(\frac{\omega - \frac{\pi}{4}}{2}\right)\right|$



(iii) $\theta = \frac{3\pi}{4}$

$$|H(\omega)| = 2\left|\cos\left(\frac{\omega-\pi}{2}\right)\right|$$