

NAME:

EE538 Digital Signal Processing I  
Exam 1Fall 2017  
Friday, Sept. 29, 2017

## Cover Sheet

Write your name on this and every page

Test Duration: 60 minutes.

Coverage: Chapters 1-5.

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **TWO** problems.

Show your work in the space provided for each problem.

You must show all work for each problem to receive full credit.

Always simplify your answers as much as possible.

Prob. No.	Topic(s)	Points
1.	Frequency Response and Interconnection of LTI Systems, Pole-Zero Diagrams DT Autocorrelation, Cross-Correlation and their Related Properties	55
2.	LTI Systems: Expressing Cross-Correlation in terms of Convolution	45

## NAME:

**Problem 1.** [55 points] Consider a DT System characterized by the difference equation below where  $j = e^{j\frac{\pi}{2}}$ .

$$\text{System: } y[n] = j y[n-1] + x[n] - x[n-4] \quad (1)$$

- (a) Draw a pole-zero diagram for this system.
- (b) Plot the magnitude of the frequency response  $|H(\omega)|$  over  $-\pi < \omega < \pi$ .
- (c) Determine the overall output  $y[n]$  of the System when the input is the sum of infinite-length sine waves below

$$x[n] = 3 + 2e^{j\frac{\pi}{2}n} + \sqrt{2}e^{-j\frac{\pi}{2}n} + e^{j\pi n} \quad (2)$$

- (d) Determine the impulse response of the system,  $h[n]$ . Write the values out in sequence form. *Hint:* the impulse response,  $h[n]$ , is finite-length.
- (e) For this same system, write an equivalent difference equation that only depends on the current input and a finite number of past inputs.
- (f) Determine the autocorrelation of the impulse response,  $r_{hh}[\ell] = h[\ell] * h^*[-\ell]$ . Write the values out in sequence form. *Hint:* You can determine the autocorrelation any way you want to, but it may be helpful to first write out in the impulse response in the form  $h[n] = e^{j\omega_0 n} \{u[n] - u[n-N]\}$  (determine  $\omega_0$  and  $N$ ) and then use a property of autocorrelation derived in class.
- (g) Determine the cross-correlation,  $r_{yx}[\ell]$ , between the output,  $y[n]$ , and the input,  $x[n]$ , when the input is the signal below, where  $p = \frac{1}{2}$ .

$$x[n] = \frac{1}{p_1} \left\{ \delta[n] + (p_1^2 - 1)p_1^n u[n] \right\} \quad (3)$$

*Handwritten notes:*  $p = \frac{1}{2}$  (with arrow pointing to  $p_1$ ),  $u$  no subscript

- (h) Determine the output,  $y[n]$ , only for the values  $n = 0, 1, 2, 3$ , when the input,  $x[n]$ , is the FINITE-length sinewave below, which has only 8 nonzero values (two periods.) That is,  $x[n]$  is only nonzero over the range  $-4 \leq n \leq 3$ . VIP, you only need to determine  $y[n]$  for four values of  $n$ . Write the output value for each these four DT time instants,  $n = 0, 1, 2, 3$ , in sequence form.

$$x[n] = e^{j\frac{\pi}{2}n} \{u[n+4] - u[n-4]\} \quad (4)$$

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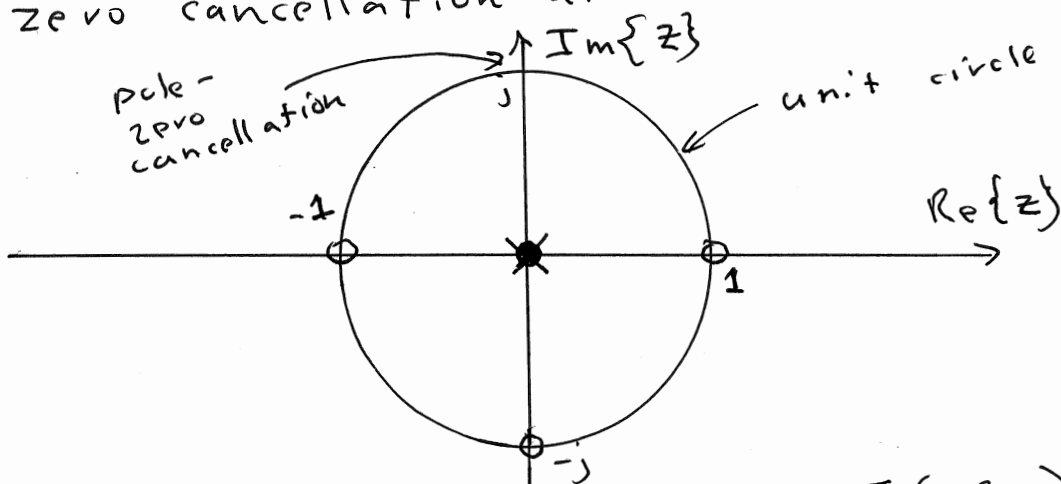
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$$(a) \quad \frac{Y(z)}{X(z)} = H(z) = \frac{1 - z^{-4}}{1 - jz^{-1}} = z^{-3} \frac{z^4 - 1}{z - j}$$

roots of  $z^4 = 1$  are  $1, -1, j, -j \Rightarrow$  zeroes

poles at  $z = j$  and  $z = 0$

pole-zero cancellation at  $z = j$



$$(b) \quad H(z) = z^{-3} (z+1)(z-1)(z+j) = z^{-3} (z^2-1)(z+j)$$

$$= z^{-3} (z^3 + jz^2 - z - j) = 1 + jz^{-1} - z^{-2} - jz^{-3}$$

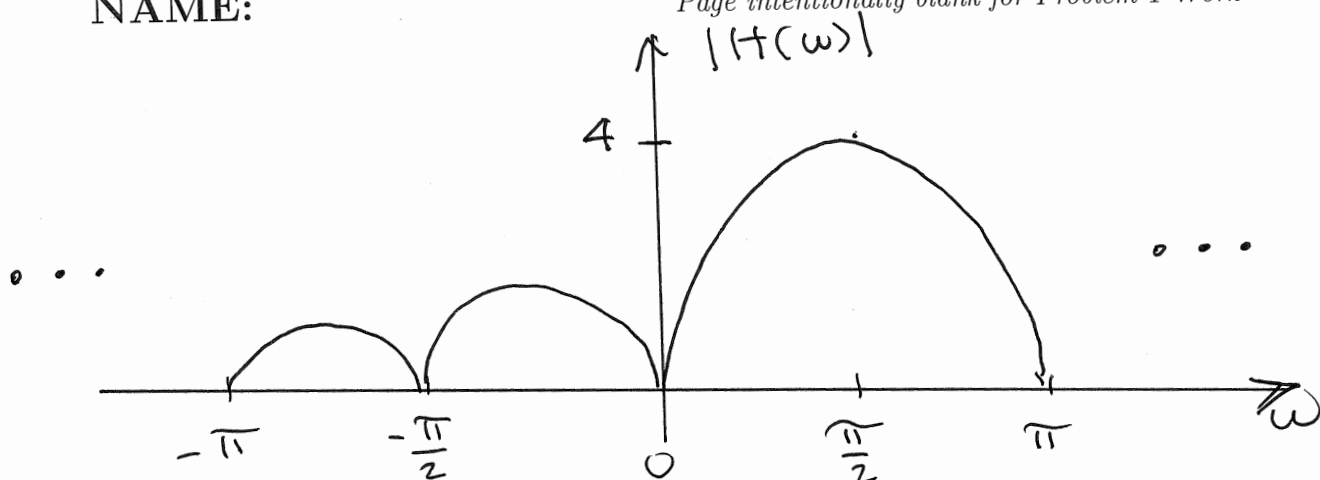
$$h[n] = \{1, j, -1, -j\} = e^{j\frac{\pi}{2}n} \{u[n] - u[n-4]\}$$

$$H(\omega) = \sum_{n=0}^3 e^{-j\omega n} = \frac{e^{-j\omega(4-1)} \sin(\frac{4}{2}\omega)}{\sin(\frac{1}{2}\omega)} \quad \omega = \omega - \frac{\pi}{2}$$

$$= e^{-j\frac{3}{2}(\omega - \frac{\pi}{2})} \frac{\sin(2(\omega - \frac{\pi}{2}))}{\sin^3(\frac{1}{2}(\omega - \frac{\pi}{2}))}$$

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$$\begin{aligned}
 (c) \quad y[n] &= 3 H(0) + 2 H\left(\frac{\pi}{2}\right) e^{j\frac{\pi}{2}n} \\
 &\quad + \sqrt{2} H\left(-\frac{\pi}{2}\right) e^{-j\frac{\pi}{2}n} + H(\pi) e^{j\pi n} \\
 &= 2(4) e^{j\frac{\pi}{2}n} = 8 e^{j\frac{\pi}{2}n}
 \end{aligned}$$

$$(d) \text{ From part (b): } h[n] = \left\{ \underset{\substack{\uparrow \\ n=0}}{1}, j, -1, -j \right\}$$

$$(e) \quad y[n] = x[n] + jx[n-1] - x[n-2] - jx[n-3]$$

$$(f) \text{ From class: if } y[n] = e^{j(\omega_0 n + \theta)} x[n]$$

$$r_{yy}[l] = e^{j\omega_0 l} r_{xx}[l]$$

$$\text{Thus: } h[n] = e^{j\omega_0 n} \{u[n] - u[n-4]\} = e^{j\frac{\pi}{2}n} \left\{ \underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, 1 \right\}$$

$$r_{hh}[l] = e^{j\frac{\pi}{2}l} \{1, 2, 3, 4, 3, 2, 1\}$$

$$= \left\{ j, -2, -3j, \underset{\substack{\uparrow \\ l=0}}{4}, 3j, -2, -j \right\}$$

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$$(g) \quad x[n] = \frac{1}{p_1} \left\{ \delta[n] + (p_1 - 1) p_1^n u[n] \right\} \quad p_1 = 1/2$$

$$r_{xx}[l] = \delta[l] \text{ as proved in class}$$

$$\begin{aligned} r_{yx}[l] &= h[l] * r_{xx}[l] \\ &= h[l] = \left\{ \begin{array}{c} 1, > j, > -j \\ \uparrow \\ l=0 \end{array} \right\} \end{aligned}$$

(h) Problem lays out region of  $y[n]$  for which there is full overlap starting at  $n=0$

$$\begin{aligned} \text{thus: } y[n] &= H\left(\frac{\pi}{2}\right) e^{j\frac{\pi}{2}n} \{u[n] - u[n-4]\} \\ &= 4 e^{j\frac{\pi}{2}n} \{u[n] - u[n-4]\} \end{aligned}$$

**NAME:** Problem 2. [45 points] Consider a complex-valued sequence

$$z[n] = x[n] + jy[n] = \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} + j(-1)^n \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \quad (5)$$

where, as indicated, the real and imaginary parts are defined below.

$$x[n] = \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \quad (6)$$

$$y[n] = (-1)^n x[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \quad (7)$$

(a) Determine the cross-correlation,  $r_{xy}[\ell]$ , between  $x[n]$  and  $y[n]$ .

Determine  $r_{yx}[\ell]$  also

(b) Determine a (closed-form) expression for the autocorrelation sequence  $r_{zz}[\ell] = z[\ell] * z^*[-\ell]$ . *Hint:* Probably easiest to work in the frequency domain.

(c) Determine a closed-form expression for the autocorrelation sequence  $r_{ww}[\ell]$  for the signal  $w[n]$  below. Is your answer the same as your answer to part (b)? Explain.

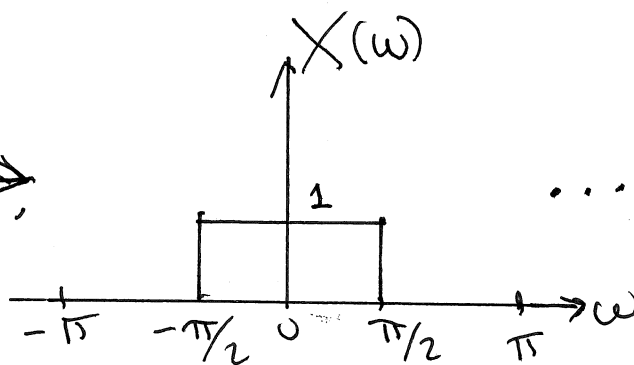
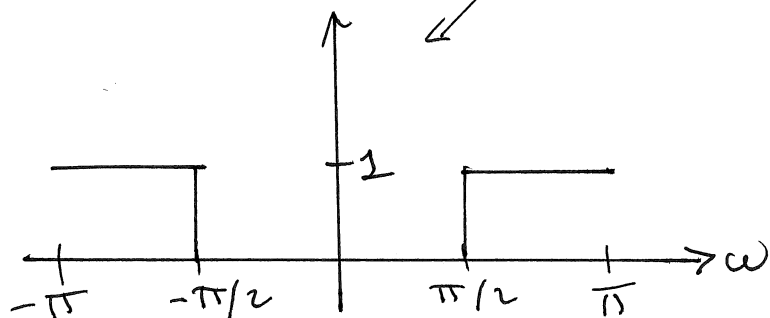
$$w[n] = e^{j\frac{\pi}{2}n} z[n] \quad (8)$$

(d) Consider a DT system with impulse response below where the input is  $z[n]$  above. Denote the output as  $\tilde{y}[n]$ , i.e.,  $\tilde{y} = z[n] * h[n]$ . Determine the cross-correlation,  $r_{\tilde{y}z}[\ell] = \tilde{y}[n] * z^*[-n]$ , between the input and output for this system.

$$h[n] = 4\delta[n-3] - 3\delta[n-4] \quad (9)$$

(a)  $r_{yx}[\ell] = y[\ell] * x^*[-\ell]$

$S_{yx}(\omega) = Y(\omega) X^*(\omega)$



$Y(\omega) = X(\omega - \pi)$

$S_{yx}(\omega) = 0$

no overlap

$r_{xy}[\ell] = x[\ell] * y^*[-\ell] \xleftrightarrow{\text{DTFT}} X(\omega) Y^*(\omega) = 0$

$= 0$

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$$\begin{aligned}(b) \quad r_{zz}[l] &= (x[l] + j y[l]) * (x^*[-l] - j y^*[-l]) \\ &= r_{xx}[l] + j \underbrace{r_{yx}[l]}_{=0} - j \underbrace{r_{xy}[l]}_{=0} + r_{yy}[l] \\ &= r_{xx}[l] + r_{yy}[l]\end{aligned}$$

$$\begin{aligned}S_{zz}(\omega) &= |X(\omega)|^2 + |Y(\omega)|^2 \\ &= 1 \quad \forall \omega\end{aligned}$$

Thus,  $r_{zz}[l] = \delta[l]$

$$(c) \quad w[n] = e^{j\frac{\pi}{2}n} z[n]$$

$$r_{ww}[l] = e^{j\frac{\pi}{2}l} r_{zz}[l] = e^{j\frac{\pi}{2}l} \delta[l]$$

$$= e^{j\frac{\pi}{2}(0)} \delta[l] = \delta[l] \quad \text{same as (b)}$$

Also:

$$S_{ww}(\omega) = Z(\omega - \frac{\pi}{2}) Z^*(\omega - \frac{\pi}{2}) = |Z(\omega - \frac{\pi}{2})|^2$$

and  $Z(\omega) = 1$  for all  $\omega$  so shifting to right by  $\frac{\pi}{2}$ , we still get 1 for all  $\omega$

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$$\begin{aligned} (d) \quad r_{\tilde{y}z}[l] &= r_{zz}[l] * h[l] = r_{zz}[l] * h[l] \\ &= \delta[l] * \{4\delta[l-3] - 3\delta[l-4]\} \\ &= 4\delta[l-3] - 3\delta[l-4] \\ &= \{ \underset{\substack{\uparrow \\ l=0}}{0}, 0, 0, \underset{\substack{\uparrow \\ l=3}}{4}, -3 \} \end{aligned}$$