538 Exam 1 Sol'n. Fall '08

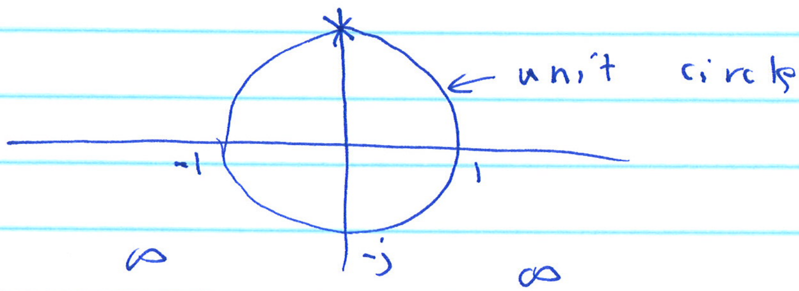
Prob. 1 $y[n] = j\,y[n-1] + x[n]$

(a) $H(z) = \dfrac{z}{z-j} \Rightarrow h[n] = (j)^n u[n]$

$= e^{j\frac{\pi}{2}n} u[n]$

- problem is identical to Exam 1 Fall 2004 Problem 1

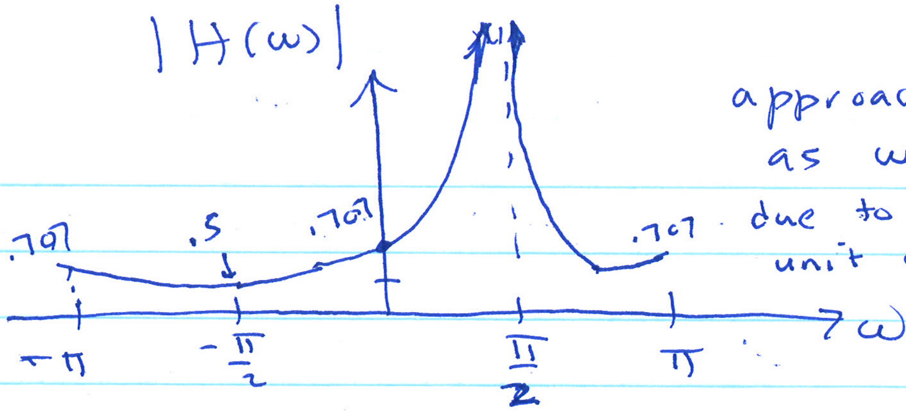
(b) not BIBO $\Rightarrow$ pole is on unit circle



Plus $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |j^n| = \sum_{n=0}^{\infty} (1) = \infty$

(c) Resonant frequency: $x[n] = e^{j\frac{\pi}{2}n} u[n]$

$y[n] = x[n] * h[n] = \underbrace{(n+1)e^{j\frac{\pi}{2}n}}_{\text{becomes unbounded as } n\to\infty} u[n]$



approaches ∞ as $\omega \to \pi/2$ due to pole on unit circle at $z = j = e^{j\frac{\pi}{2}}$

(e) $x[n] = e^{j0n} + e^{-j\frac{\pi}{2}n} + e^{j\pi n}$

$$y[n] = H(0)e^{j0n} + H(-\tfrac{\pi}{2})e^{-j\frac{\pi}{2}} + H(\pi)e^{j\pi n}$$

$$H(\omega) = H(z) = \frac{1}{1-jz^{-1}}\Bigg|_{z=e^{j\omega}} = \frac{1}{1-e^{j(\omega-\pi/2)}}$$

$$H(0) = \frac{1}{1-j} \Rightarrow |H(0)| = \frac{1}{\sqrt{2}}$$

$$H(-\tfrac{\pi}{2}) = \frac{1}{2}$$

$$H(\pi) = \frac{1}{1+j} \Rightarrow |H(\pi)| = \frac{1}{\sqrt{2}}$$

Prob. 2 Solution

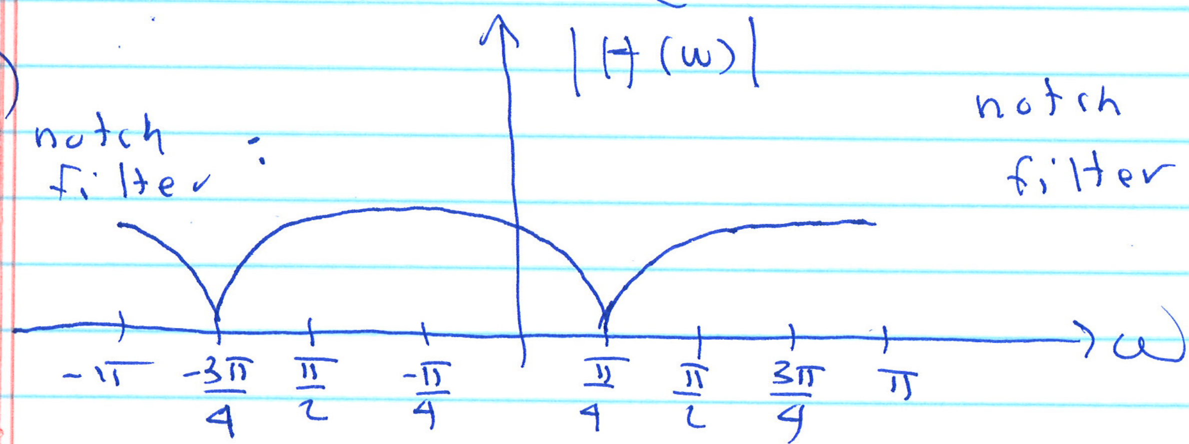
$$y[n] = \frac{1}{4}j\, y[n-2] + x[n] - j\, x[n-2]$$

(a) $$H(z) = \frac{z^2 - j}{z^2 - \frac{1}{4}j} = \frac{\left(z - e^{j\frac{\pi}{4}}\right)\left(z + e^{j\frac{\pi}{4}}\right)}{\left(z - \frac{1}{2}e^{j\frac{\pi}{4}}\right)\left(z + \frac{1}{2}e^{j\pi/4}\right)}$$

notch at $\omega = \frac{\pi}{4}$ and $\omega = -\frac{3\pi}{4}$ (see below)

$$-e^{j\frac{\pi}{4}} = e^{j\left(\frac{\pi}{4} - \pi\right)} = e^{-j\frac{3\pi}{4}}$$

(b)



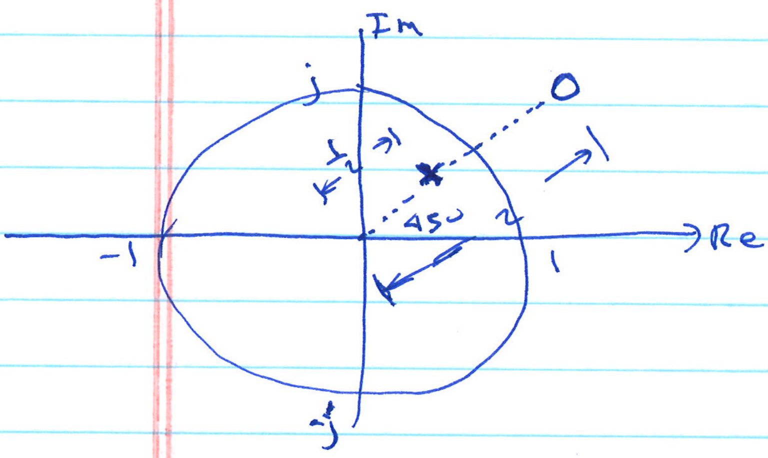
(c) easy to show:

$$\frac{1}{2}\frac{\left(z - 2e^{j\frac{\pi}{4}}\right)}{\left(z - \frac{1}{2}e^{j\frac{\pi}{4}}\right)} + \frac{1}{2}\frac{\left(z + 2e^{j\frac{\pi}{4}}\right)}{\left(z + \frac{1}{2}e^{j\frac{\pi}{4}}\right)}$$

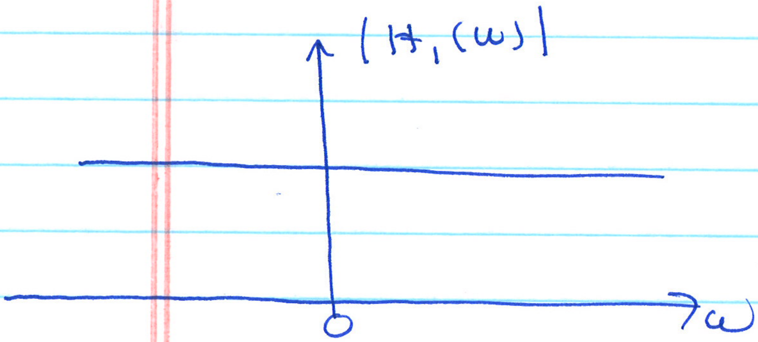
$$= \frac{z^2 - j}{z^2 - \frac{1}{4}j}$$

Thus, both systems are all-pass filters

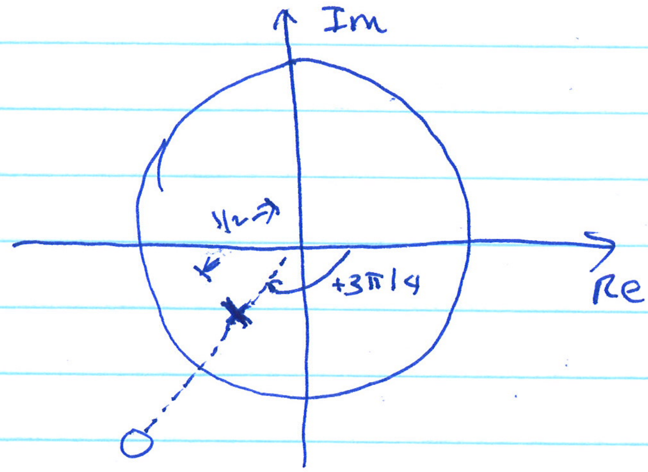
System 1



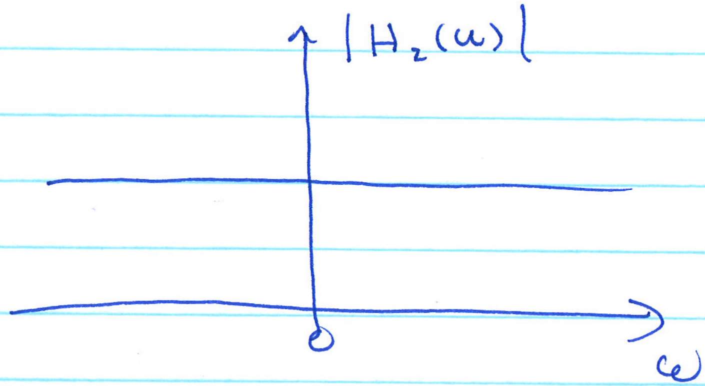
ROC: $|z| > \frac{1}{2}$



System 2



ROC: $|z| > 1/2$



Solution to Prob. 3

$r_{xx}[0] = 4^2 + 2^2 + 1^2 = 21$

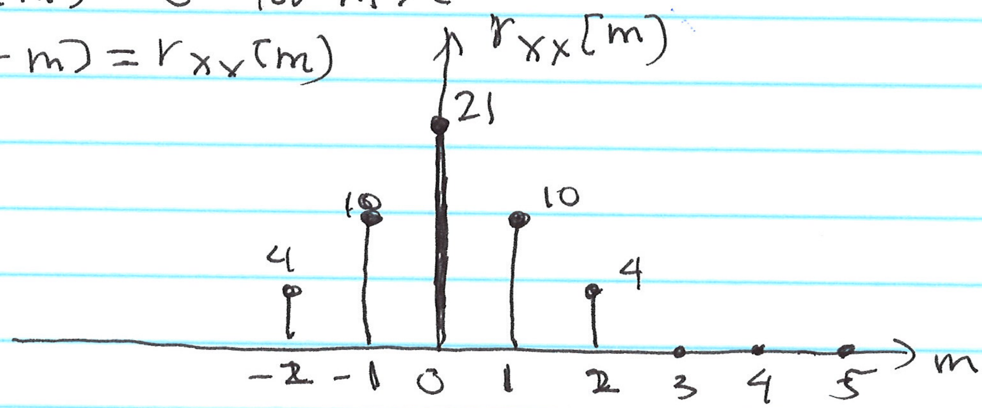
$$r_{xx}[1] = \left.\begin{matrix} 4 & 2 & 1 & \\ & 4 & 2 & 1 \end{matrix}\right\} \; 2\cdot 4 + 1\cdot 2 = 10$$

$$r_{xx}[2] = \left.\begin{matrix} 4 & 2 & 1 & & \\ & & 4 & 2 & 1 \end{matrix}\right\} = 1\cdot 4 = 4$$

$r_{xx}[m] = 0$ for $m > 2$

$r_{xx}[-m] = r_{xx}[m]$

(a)



(b)

$$h[n] = 2\delta[n-2] + \delta[n-6]$$

$$r_{yx}[m] = r_{xx}[m] * h[m] =$$

$$= r_{xx}[m] * \{2\delta[n-2] + \delta[n-6]\}$$

$$= 2r_{xx}[m-2] + r_{xx}[m-6]$$