NAME:

EE538 Digital Signal Processing I — Fall 2016
Exam 1 — Friday, Sept. 30, 2016

Cover Sheet

Write your name on this and every page
Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
Show your work in the space provided for each problem.
You must show all work for each problem to receive full credit.
Always simplify your answers as much as possible.

Prob. No.	Topic(s)	Points
1.	Frequency Response and Interconnection of LTI Systems, Pole-Zero Diagrams	35
2.	DT Autocorrelation, Cross-Correlation and their Related Properties	30
3.	LTI Systems: Expressing Cross-Correlation in terms of Convolution	35

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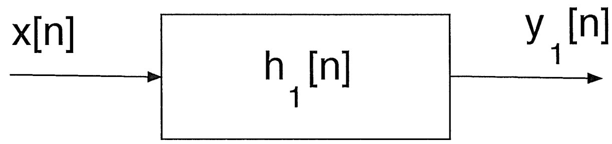
Problem 1. [35 points]

(a) Consider System 1 characterized by the difference equation

$$\text{System 1:} \qquad y_1[n] = -\frac{9}{16}y_1[n-2] + \frac{9}{16}x[n] + x[n-2]$$

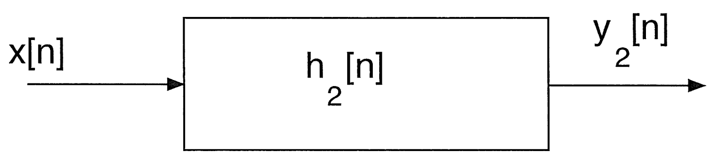


(i) Draw a pole-zero diagram for this system.

(ii) Plot the magnitude of the frequency response $|H_1(\omega)|$ over $-\pi < \omega < \pi$.

(b) Consider System 2 with impulse response $h_2[n]$ below:

$$\text{System 2:} \qquad y_2[n] = -\frac{9}{16}y_2[n-2] + x[n] + x[n-2]$$



(i) Draw a pole-zero diagram for this system.

(ii) Plot the magnitude of the frequency response $|H_2(\omega)|$ over $-\pi < \omega < \pi$.

(c) Determine the overall output $y[n]$ of the parallel combination of System 2 when the input is:

$$x[n] = 3 + 2\cos\left(\frac{\pi}{2}n\right) + \cos(\pi n)$$

System 1

Prob. 1 (a) $y_1[n] = -\frac{9}{16} y_1[n-2] + \frac{9}{16} x[n] + x[n-2]$

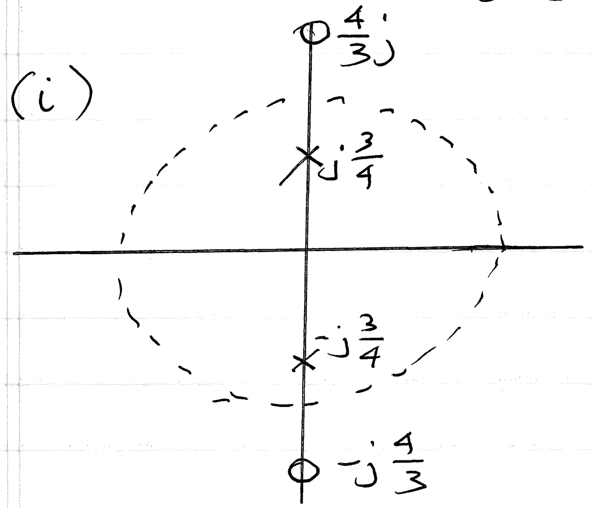
ZT of both sides

$$Y_1(z)\left\{1 + \frac{9}{16} z^{-2}\right\} = \left(\frac{9}{16} + z^{-2}\right) X(z)$$

$$H_1(z) = \frac{\frac{9}{16} + z^{-2}}{1 + \frac{9}{16} z^{-2}} = \frac{z^2 \frac{9}{16} + 1}{z^2 + \frac{9}{16}} = \frac{9}{16} \frac{\left(z^2 + \frac{16}{9}\right)}{z^2 + \frac{9}{16}}$$

Zeroes: $z = \pm j \frac{4}{3}$ poles: $z = \pm j \frac{3}{4}$

(i)

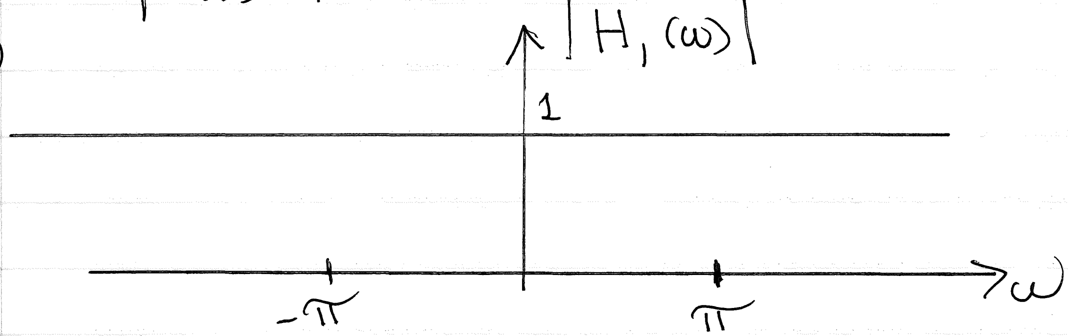


for $\omega = 0 \Rightarrow z = 1$:

$$\frac{\frac{9}{16} + 1}{1 + \frac{9}{16}} = 1$$

• all-pass filter:

(ii)



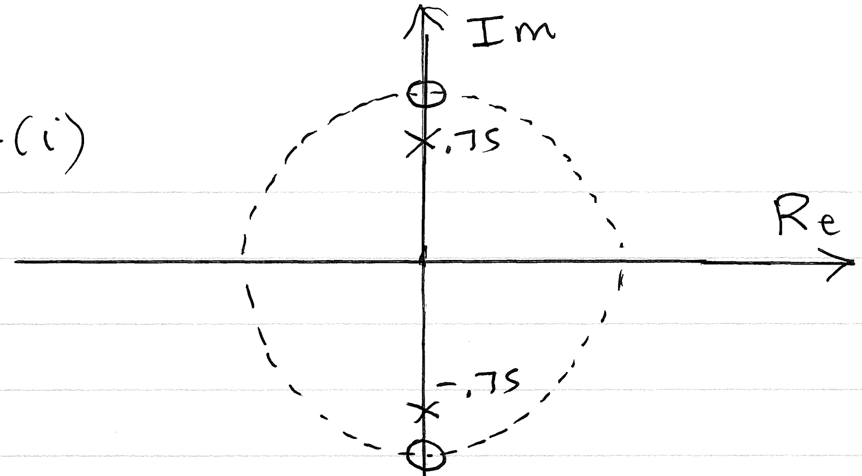
(b) System 2: $y_2[n] = -\frac{9}{16} y_2[n-2] + x[n] + x[n-2]$

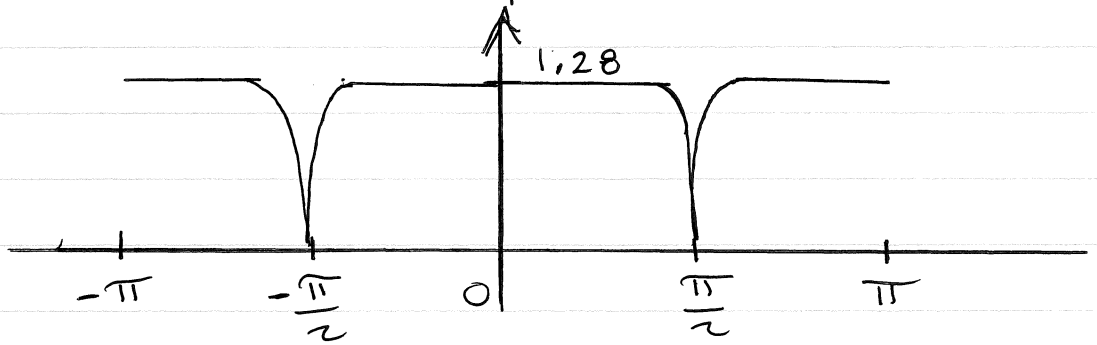
$$H_2(z) = \frac{1 + z^{-2}}{1 + \frac{9}{16} z^{-2}} = \frac{z^2 + 1}{z^2 + \frac{9}{16}}$$

$j, -j$ zeroes

$\frac{3}{4}j, -\frac{3}{4}j$ poles

(b)-(i)





$$H_2(z) = \frac{1+z^{-2}}{1+\frac{9}{16}z^{-2}} = \frac{z^2+1}{z^2+\frac{9}{16}}$$

$$H_2(\omega)\Big|_{\omega=0} = H_{2_{ZT}}(z)\Big|_{z=1} = \frac{1+1}{1+\frac{9}{16}} = \frac{2}{\frac{25}{16}} = \frac{32}{25}$$

$$H_2(\omega)\Big|_{\omega=\pi} = H_{2_{ZT}}(z)\Big|_{z=-1} = \frac{32}{25} = 1+\frac{7}{25} = 1.28$$

(c) $$x[n] = 3e^{j0\cdot n} + \underbrace{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}_{\text{notched out}} + e^{j\pi n}$$

$$y[n] = \frac{32}{25}(3)e^{j0\cdot n} + \frac{32}{25}(-1)^n$$

<u>note:</u> $\cos(\pi n) = (-1)^n = e^{j\pi n} = e^{-j\pi n}$

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Problem 2. [35 points]

(a) Determine the autocorrelation $r_{x_0x_0}[\ell]$ of the length-4 sequence $x_0[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell = 0$ is located. Briefly indicate which properties of autocorrelation you use to solve each part.

$$x_0[n] = \{1, 2, 3, 4\} = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$

(b) Determine the autocorrelation $r_{x_1x_1}[\ell]$ of the sequence $x_1[n]$ below defined in terms of $x_0[n]$ in part (a). Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_1[n] = e^{j\left(\frac{\pi}{2}(n-2)\right)} x_0[n-2]$$

(c) Determine the autocorrelation $r_{x_2x_2}[\ell]$ of the sequence $x_2[n]$ below defined in terms of $x_0[n]$ in part (a). Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_2[n] = e^{j(\pi n + \sqrt{\pi})} x_0[n]$$

(d) Determine the autocorrelation $r_{x_3x_3}[\ell]$ of the sequence $x_3[n]$ below defined in terms of $x_0[n]$ in part (a). Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x_3[n] = e^{-j\frac{\pi}{2}n} x_0^*[-n]$$

(e) Sum your answers to parts (a) thru (d) to form the sum below. Do a stem plot of $r_{xx}[\ell]$.

$$r_{xx}[\ell] = r_{x_0x_0}[\ell] + r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell] + r_{x_3x_3}[\ell]$$

Problem 2

$x_0[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 4\}$

(a) $r_{x_0 x_0}[\ell] = \{1, 2, 3, 4\} * \{4, 3, 2, 1\}$

4	3	2	1			
	8	6	4	2		
		12	9	6	3	
			16	12	8	4
4	11	20	30	20	11	4

$r_{x_0 x_0}[\ell] = \{4, 11, 20, \underset{\substack{\uparrow \\ \ell=0}}{30}, 20, 11, 4\}$

(b) $x_1[n] = e^{j\frac{\pi}{2}(n-2)} x_0[n-2]$

$\tilde{x}_1[n] = x_1[n+2]$ has same auto correlation

$= e^{j\frac{\pi}{2}n} x_0(n)$ sequence as $x_1(n)$

$r_{x_1 x_1}[\ell] = \underbrace{e^{j\frac{\pi}{2}n}}_{(j)^n} r_{x_0 x_0}(\ell)$

I mixed n and ell here: the n's should be ell's

$n=0$: 1 $\quad$ $n=1$: j $\quad$ $n=+2$: -1 $\quad$ $n=+3$: $-j$

$r_{x_1 x_1}[\ell] = \{+4j, -11, -20j, \underset{\substack{\uparrow \\ \ell=0}}{30}, +20j, -11, -4j\}$

Prob. 2(c) $x_2[n] = e^{j(\pi n + \sqrt{\pi})} x_0[n]$

$r_{x_2x_2}[\ell] = e^{j\pi\ell}\, r_{x_0x_0}[\ell] = (-1)^{\ell}\, r_{x_0x_0}[\ell]$

$$r_{x_2x_2}[\ell] = \{-4,\ 11,\ -20,\ \underset{\substack{\uparrow \\ \ell=0}}{30},\ -20,\ 11,\ -4\}$$

Prob. 2(d) $x_3[n] = e^{-j\frac{\pi}{2}n}\, x_0^*[-n]$

Since $x_0^*[-n]$ has the same autocorrelation sequence

$r_{x_3x_3}[\ell] = e^{-j\frac{\pi}{2}\ell}\, r_{x_0x_0}[\ell] = (-j)^{\ell}\, r_{x_0x_0}[\ell]$

$$= \{-4j,\ -11,\ 20j,\ 30,\ -20j,\ -11,\ 4j\}$$

(e) $r_{xx}[\ell] = \sum_{k=0}^{3} r_{x_kx_k}[\ell] = 4 \cdot 30\ \delta[\ell]$

$$= 120\ \delta[\ell]$$

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Problem 3. [25 points]

(a) Determine the autocorrelation $r_{xx}[\ell]$ of the length-4 sequence $x[n]$ below, which is written two different ways. Write your answer in sequence form indicating where the value for $\ell = 0$ is located.

$$x[n] = \{1, -1, 1, 1\} = \delta[n] - \delta[n-1] + \delta[n-2] + \delta[n-3]$$

(b) The sequence $x[n]$ defined above is input to the system described by the simple difference equation below. Determine and write an equation for the cross-correlation, $r_{yx}[\ell]$, between the output and input in terms of the autocorrelation of the input, $r_{xx}[\ell]$.

$$y[n] = 4x[n-3] + x[n-4]$$

(c) Compute the numerical values of the cross-correlation $r_{yx}[\ell]$ between the output and input and do a stem plot OR write out the values in sequence form clearly indicating the point corresponding to $\ell = 0$.

(d) What is the value of $r_{yx}[\ell]$ at $\ell = 3$? What is the value of $r_{yx}[\ell]$ at $\ell = 4$? Does the cross-correlation $r_{yx}[\ell]$ exhibit a peak at each of the two delays? Briefly explain.

Problem 3 $x[n] = \{1, -1, 1, 1\}$

(a)

$$r_{xx}[\ell] = x[\ell] * x^*[-\ell]$$
$$= \{1, -1, 1, 1\} * \{1, 1, -1, 1\}$$

1	1	-1	1			
	-1	-1	1	-1		
		1	+1	-1	1	
			1	1	-1	1
1	0	-1	4	-1	0	1

$$r_{xx}[\ell] = \{1, 0, -1, \underset{\substack{\uparrow \\ \ell=0}}{4}, -1, 0, 1\}$$

(b)

$$y[n] = 4x[n-3] + x[n-4]$$
$$h[n] = 4\delta[n-3] + \delta[n-4]$$
$$r_{yx}[\ell] = h[\ell] * r_{xx}[\ell]$$
$$= 4r_{xx}[\ell-3] + r_{xx}[\ell-4]$$

(c) $\{\underset{\substack{\uparrow \\ \ell=0}}{0}, 0, 0, 4, 1\} * \{1, 0, -1, \underset{\substack{\uparrow \\ \ell=0}}{4}, -1, 0, 1\}$

$$\begin{array}{cccccccc}
4, & 0, & -4, & 16, & -4, & 0, & 4 & \\
 & 1, & 0, & -1, & 4, & -1, & 0, & 1 \\
\hline
\underset{\substack{\uparrow \\ \ell=0}}{4}, & 1, & -4, & \underset{\substack{\uparrow \\ \ell=3}}{15}, & 0, & -1, & 4, & 1
\end{array}$$

Problem 3 (c)

$$r_{yx}[\ell] = \{4, 1, -4, 15, 0, -1, 4, 1\}$$

$\ell=0$ $\ell=3$ $\ell=4$

(d) $r_{xy}[3] = 15$ peak value

$r_{xy}[4] = 0$ despite the delay of 4 ($x[n-4]$)

This is due to the "sidelobes" of $r_{xx}[\ell]$ and the fact that the delayed replica at 3 has an amplitude 4x that of the delayed replica at delay 4