Cover Sheet

Test Duration: 55 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done on blank 8 1/2 x 11 sheets.
Please have your site coordinator scan in your work sheets
and email pdf file to me at michael.zoltowski@gmail.com
Or FAX to me at 765-494-3358.
You must show all work for each problem to receive full credit.
Do **not** return the exam itself; just your work on separate sheets.

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<th>Prob. No.</th>
<th>Topic(s)</th>
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<td>1.</td>
<td>LTI Systems: Properties, Transfer Functions, Frequency Response</td>
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<td>2.</td>
<td>Interconnection of LTI Systems: Transfer Functions, Frequency Response</td>
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<td>3.</td>
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Point Breakdown
Problem 1. [35 points]
Consider a causal DT LTI system with impulse response
\[ y[n] = e^{j\frac{\pi}{2}} y[n - 1] + x[n] \]

(a) Find the system transfer function \( H(z) \) of this system and draw the pole-zero diagram.

(b) Is the system BIBO stable? Substantiate your answer.

(c) Find a bounded input signal \( x[n] \) that produces an unbounded output from this system.

(d) Plot a rough sketch of the magnitude of the DTFT of \( h[n] \), \( |H(\omega)| \), over \(-\pi < \omega < \pi\), showing as much detail as possible.

(e) Determine the output \( y[n] \) for the following input:
\[ x[n] = 1 + (-j)^n + (-1)^n \]
Problem 2. [35 points]
Consider the causal, second-order LTI system described by the difference equation below.

\[ y[n] = 0.25j \ y[n - 2] + x[n] - jx[n - 2] \]

(a) Find the system transfer function \( H(z) \) of this system and draw the pole-zero diagram.

(b) Plot the magnitude, \( |H(\omega)| \), of the DTFT of the impulse response of the system over \(-\pi < \omega < \pi\), showing as much detail as possible. In particular, explicitly point out if there are any values of \( \omega \) for which \( |H_i(\omega)| \) is exactly zero.

(c) Consider implementing the second-order difference equation above as two first-order systems (one pole each) in parallel as shown in the diagram.

![Diagram showing two first-order systems in parallel](image)

The upper first-order system has impulse response \( h_1[n] \) and is described by the difference equation

\[ y_1[n] = a_1^{(1)} \ y_1[n - 1] + b_0^{(1)} \ x[n] + b_1^{(1)} \ x[n - 1] \]

The lower first-order system has impulse response \( h_2[n] \) and is described by the difference equation

\[ y_2[n] = a_1^{(2)} \ y_2[n - 1] + b_0^{(2)} \ x[n] + b_1^{(2)} \ x[n - 1] \]

Determine the numerical values of \( a_1^{(i)} \), \( b_0^{(i)} \), and \( b_1^{(i)} \), \( i = 1, 2 \) – six values total. **NOTE:** Each of the two first-order systems has a single non-zero zero and a single non-zero pole. In order to get a unique answer, you are given that \( a_0^{(1)} = \frac{1}{2} \), and you must find 5 numerical values: \( a_1^{(1)}, b_0^{(1)}, a_1^{(2)}, b_0^{(2)}, \) and \( b_1^{(2)} \).

(d) For EACH of the two first-order systems, \( i = 1, 2 \), do the following:

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for \( H_i(z) \).

(iii) Determine the DTFT of \( h_i[n] \) and plot the magnitude \( |H_i(\omega)| \) over the interval \(-\pi < \omega < \pi\) showing as much detail as possible.
Problem 3. [30 points]
Consider the DT signal below which is only nonzero for three values of $n$:

$$x[n] = 4\delta[n] + 2\delta[n - 1] + \delta[n - 2]$$

(i) Compute and plot the autocorrelation, $r_{xx}[\ell]$, of $x[n]$.

(ii) $x[n]$ is passed through a DT linear system characterized by the difference equation below:

$$y[n] = 2x[n - 2] + x[n - 6]$$

Compute and plot the cross-correlation, $r_{yx}[\ell]$, between the input $x[n]$ and output $y[n]$. 

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