# EE538 Digital Signal Processing I — Session 14
# Exam 1 — Live: Fri., Sept. 26, 2008

## Cover Sheet

Test Duration: 55 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done on blank 8 1/2 x 11 sheets.
Please have your site coordinator scan in your work sheets
and email pdf file to me at michael.zoltowski@gmail.com
Or FAX to me at 765-494-3358.
You must show all work for each problem to receive full credit.
Do **not** return the exam itself; just your work on separate sheets.

| **Prob. No.** | **Topic(s)** | **Points** |
|---|---|---|
| 1. | LTI Systems: Properties, Transfer Functions, Frequency Response | 35 |
| 2. | Interconnection of LTI Systems: Transfer Functions, Frequency Response | 35 |
| 3. | DT Autocorrelation, Cross-Correlation | 30 |

Point Breakdown

**Problem 1.** [35 points]

Consider a causal DT LTI system with impulse response

$$y[n] = e^{j\frac{\pi}{2}} y[n-1] + x[n]$$

7 (a) Find the system transfer function $H(z)$ of this system and draw the pole-zero diagram.

7 (b) Is the system BIBO stable? Substantiate your answer.

7 (c) Find a bounded input signal $x[n]$ that produces an unbounded output from this system.

7 (d) Plot a rough sketch of the magnitude of the DTFT of $h[n]$, $|H(\omega)|$, over $-\pi < \omega < \pi$, showing as much detail as possible.

7 (e) Determine the output $y[n]$ for the following input:

$$x[n] = 1 + (-j)^n + (-1)^n$$

**Problem 2.** [35 points]

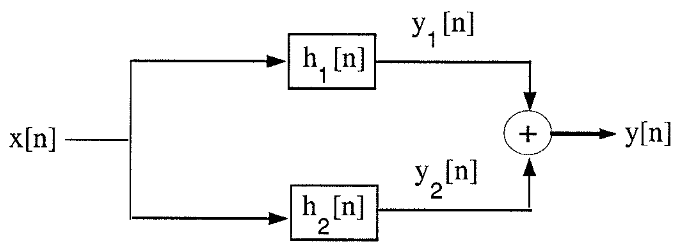
Consider the causal, second-order LTI system described by the difference equation below.

$$y[n] = 0.25j\ y[n-2] + x[n] - jx[n-2]$$

5 (a) Find the system transfer function $H(z)$ of this system and draw the pole-zero diagram.

5 (b) Plot the magnitude, $|H(\omega)|$, of the DTFT of the impulse response of the system over $-\pi < \omega < \pi$, showing as much detail as possible. In particular, explicitly point out if there are any values of $\omega$ for which $|H_i(\omega)|$ is exactly zero.

10 (c) Consider implementing the second-order difference equation above as two first-order systems (one pole each) in parallel as shown in the diagram.




The upper first-order system has impulse response $h_1[n]$ and is described by the difference equation

$$y_1[n] = a_1^{(1)}\ y_1[n-1] + b_0^{(1)}x[n] + b_1^{(1)}\ x[n-1]$$

The lower first-order system has impulse response $h_2[n]$ and is described by the difference equation

$$y_2[n] = a_1^{(2)}\ y_2[n-1] + b_0^{(2)}x[n] + b_1^{(2)}\ x[n-1]$$

Determine the numerical values of $a_1^{(i)}$, $b_0^{(i)}$, and $b_1^{(i)}$, $i = 1, 2$ – six values total. **NOTE:** Each of the two first-order systems has a single non-zero zero and a single non-zero pole. In order to get a unique answer, you are given that $b_0^{(1)} = \frac{1}{2}$, and you must find 5 numerical values: $a_1^{(1)}$, $b_1^{(1)}$, $a_1^{(2)}$, $b_0^{(2)}$, and $b_1^{(2)}$.

(d) For EACH of the two first-order systems, $i = 1, 2$, do the following:

5 (i) Plot the pole-zero diagram.

5 (ii) State and plot the region of convergence for $H_i(z)$.

5 (iii) Determine the DTFT of $h_i[n]$ and plot the magnitude $|H_i(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible.

**Problem 3.** [30 points]

Consider the DT signal below which is only nonzero for three values of $n$:

$$x[n] = 4\delta[n] + 2\delta[n-1] + \delta[n-2] \tag{1}$$

/5 (i) Compute and plot the autocorrelation, $r_{xx}[\ell]$, of $x[n]$.

15 (ii) $x[n]$ is passed through a DT linear system characterized by the difference equation below:

$$y[n] = 2x[n-2] + x[n-6]$$

Compute and plot the cross-correlation, $r_{yx}[\ell]$, between the input $x[n]$ and output $y[n]$.