Solution to Prob. 1;

(a) Time-shift doesn't affect autocorrelation

$$r_{y_2 y_2}[\ell] = r_{x_2 x_2}[\ell]$$

Hence, sequences are still complementary, i.e., $y_1(n)$ and $y_2(n)$ are complementary

(b) $x[n]$ and $x^*[-n]$ have the same autocorrelation due to commutativity of convolution $\Rightarrow r_{y_1 y_1}[\ell] = r_{x_1 x_1}[\ell]$

$\Rightarrow y_1[n]$ and $y_2[n]$ are complementary.

(c) $r_{y_1 y_1}[\ell] + r_{y_2 y_2}[\ell] =$

$$e^{j\omega_0 \ell}\{r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell]\}$$

$$= e^{j\omega_0 \ell}\, c\, \delta[\ell] = e^{j0}\, c\, \delta[\ell]$$

$$= c\, \delta[n]$$

$\Rightarrow y_1[n]$ and $y_2[n]$ are complementary

Prob. 1 Soln (cont.)

(d) $y_1[n] = \frac{1}{2}\left(x_1[n] + x_2[n]\right)$

$y_2[n] = \frac{1}{2}\left\{x_1[n] - x_2[n]\right\}$

$$r_{y_1y_1}[\ell] = \frac{1}{4}\left\{r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell]\right\} + \frac{1}{4} r_{x_2x_1}[\ell] + \frac{1}{4} r_{x_1x_2}[\ell]$$

$$r_{y_2y_2}[\ell] = \frac{1}{4}\left\{r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell]\right\} + \frac{1}{4} r_{x_1x_2}[\ell] - \frac{1}{4} r_{x_2x_1}[\ell]$$

$$r_{y_1y_1}[\ell] + r_{y_2y_2}[\ell] = \frac{1}{2} c\,\delta[\ell]$$

$\Rightarrow$ complementary? Yes!

But now there are 3 possible values for either $y_1[n]$ and $y_2[n]$: $\in \{-1, 0, 1\}$

$\Rightarrow$ so <u>not</u> unimodular

Some values are zero

Prob. 1 soln (cont.)

(e) Starting point:

$y_1[n]$ and $y_2[n]$ are complementary

Thus: $r_{z_1 z_1}[\ell] + r_{z_2 z_2}[\ell]$

$$= 2\left\{ r_{y_1 y_1}[\ell] + r_{y_2 y_2}[\ell] \right\}$$

$$+ r_{y_1 y_2}[\ell] + r_{y_2 y_1}[\ell]$$

$$- r_{y_1 y_2}[\ell] - r_{y_1 y_2}[\ell]$$

$$= \mathcal{E}\,\delta[\ell]$$

$\Rightarrow$ complementary? Yes!

And now $z_1[n]$ and $z_2[n]$ are sequences of +1's and -1's

wherever $y_1[n] = 1$ or $-1 \Rightarrow y_2[n] = 0$

$y_2[n] = 1$ or $-1 \Rightarrow y_1[n] = 0$

From Fall 2013 Exam 1, we know:

(f)

$$r_{y_1 y_1}[\ell] = r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell]$$

$$+ r_{x_1 x_2}[\ell+N] + r_{x_2 x_1}[\ell-N]$$

$$r_{y_2 y_2}[\ell] = r_{x_1 x_1}[\ell] + r_{x_2 x_2}[\ell]$$

$$- r_{x_1 x_2}[\ell+N] - r_{x_2 x_1}[\ell-N]$$

Sum is: $2c\,\delta[\ell] = r_{y_1 y_1}[\ell] + r_{y_2 y_2}[\ell]$

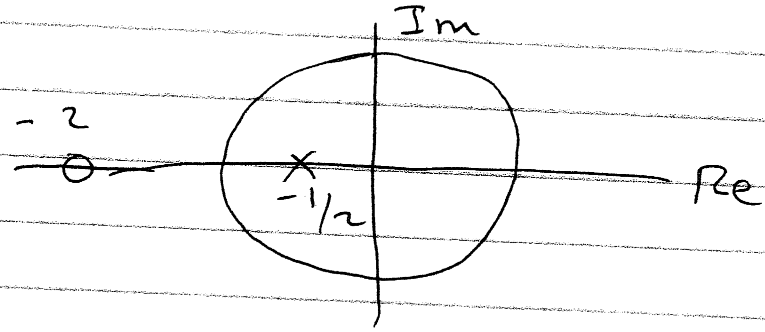
- The sequences do not overlap and are right up against each other

$$\left.\begin{array}{l} y_1[n] \text{ is unimodular} \\ y_2[n] \text{ is unimodular} \end{array}\right\} \text{sequences of } +1\text{'s and } -1\text{'s}$$

Prob. 2 (soln) Solution

(a)
$$\frac{Y(z)}{X(z)} = \frac{\frac{1}{2} + z^{-1}}{2 + \frac{1}{2} z^{-1}} \cdot \frac{z}{z} = \frac{\frac{1}{2} z + 1}{z + \frac{1}{2}}$$

$$\left.\begin{array}{ll} \text{zero at} & z = -2 \\ \text{pole at} & z = -\frac{1}{2} \end{array}\right\} \text{all-pass filter}$$



(b)
$$\frac{1}{2} \frac{z}{z+\frac{1}{2}} + z^{-1} \frac{z}{z+1/2}$$

note:
$$\left(-\frac{1}{2}\right)^n \left(-\frac{1}{2}\right)^{-1} = -2 \left(-\frac{1}{2}\right)^n$$

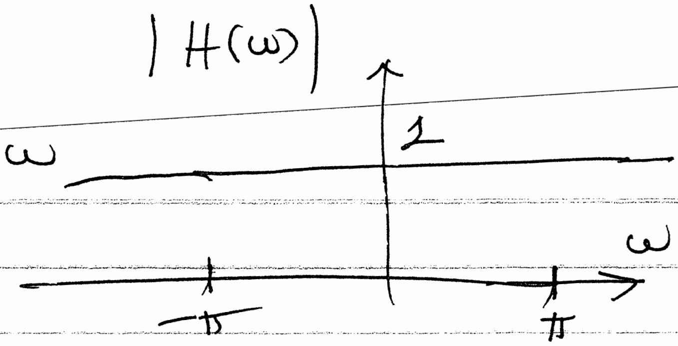
$$\Leftrightarrow h[n] = \frac{1}{2}\left(-\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^{n-1} u[n-1]$$

$$= \frac{1}{2} \delta[n] + \left(\frac{1}{2} - 2\right)\left(-\frac{1}{2}\right)^n u[n-1]$$

$$= \frac{1}{2} \delta[n] - \frac{3}{2}\left(-\frac{1}{2}\right)^n u[n-1]$$

$$= +2 \left\{ \delta[n] + \left(-\frac{3}{4}\right)\left(-\frac{1}{2}\right)^n u[n] \right\}$$

(c) $|H(\omega)| = 1 \quad \forall \omega$



(d) $\angle H(\omega)$ is nonlinear

but $|H(\omega)| = 1 \quad \forall \omega$ thus:

$A_0 = 2 \qquad A_1 = 1 \qquad A_2 = \sqrt{2} \qquad A_3 = 3$

amplitudes of the sinewaves are unchanged ⇒ as they pass thru all-pass filter

BUT phases change ⇒

$$y[n] \neq x[n]$$

(e) $x[n] = \frac{1}{p}\left\{\delta[n] + (p^2-1)\, p^n u[n]\right\}$

⇒ all-pass signal

$r_{xx}[\ell] = \delta[\ell]$

(f) $r_{yx}[\ell] = h[\ell] * \overbrace{r_{xx}[\ell]}^{\delta[\ell]}$

$= h[\ell] = 2\left\{\delta[\ell] - \frac{3}{4}\left(-\frac{1}{2}\right)^{\ell} u[\ell]\right\}$

(c)

$$r_{yy}[l] = r_{xx}[l] * \underbrace{r_{hh}[l]}_{\delta[l]}$$

$$= r_{xx}[l]$$

1	1	1	-1	1	
1	1	1	-1	1	$l=0 \Rightarrow 5$
	1	1	1	-1	$l=1 \Rightarrow 0$
		1	1	1	$l=2 \Rightarrow 1$
			1	1	$l=3 \Rightarrow 0$
				1	$l=4 \Rightarrow 1$

$$r_{yy}[l] = r_{xx}[l]$$

$$= \{1, 0, 1, 0, \underset{\substack{\uparrow \\ l=0}}{5}, 0, 1, 0, 1\}$$