Solution to Prob. 1:

(a) Time-shift doesn't affect autocorrelation

\[ r_{y_1 y_2}[l] = r_{x_2 x_2}[l] \]

Hence, sequences are still complementary, i.e., \( y_1[n] \) and \( y_2[n] \) are complementary.

(b) \( x[n] \) and \( x^*[\cdot-n] \) have the same autocorrelation due to commutativity of convolution \( \Rightarrow r_{y_1 y_1}[l] = r_{x_1 x_1}[l] \)

\( \Rightarrow y_1[n] \) and \( y_2[n] \) are complementary.

(c) \[ r_{y_1 y_1}[l] + r_{y_2 y_2}[l] = e^{i\omega_0 l} \{ r_{x_1 x_1}[l] + r_{x_2 x_2}[l] \} \]

\[ = e^{i\omega_0 l} c \cdot \delta[l] = e^{i\omega_0 n} c \cdot \delta[n] \]

\( \Rightarrow y_1[n] \) and \( y_2[n] \) are complementary.
Problem 1: Solution (cont.)

(c) \[ y_1(n) = \frac{1}{2} (x_1(n) + x_2(n)) \]
\[ y_2(n) = \frac{1}{2} (x_1(n) - x_2(n)) \]

\[ r_{y_1y_1}[\ell] = \frac{1}{4} \{ r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell] \} \]
\[ + \frac{1}{4} r_{x_2x_1}[\ell] + \frac{1}{4} r_{x_1x_2}[\ell] \]

\[ r_{y_1y_2}[\ell] = \frac{1}{4} \{ r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell] \} \]

\[ - \frac{1}{4} r_{x_1x_2}[\ell] - \frac{1}{4} r_{x_2x_1}[\ell] \]

\[ r_{y_1y_1}[\ell] + r_{y_2y_2}[\ell] = \frac{1}{2} c_0 \delta[\ell] \]

\( \Rightarrow \) complementary? Yes!

But now there are 3 possible values for either \( y_1(n) \) and \( y_2(n) \): \( \mathbb{E} \{-1, 0, 1\} \)

\( \Rightarrow \) so not unimodular

Some values are zero
(e) Starting point:

\[ y_1(n) \text{ and } y_2(n) \text{ are complementary} \]

Thus:

\[
\begin{align*}
R_{z_1z_1}(e) + R_{z_2z_2}(e) & = 2 \left( R_{y_1y_1}(e) + R_{y_2y_2}(e) \right) \\
& \quad + R_{y_1y_2}(e) + R_{y_2y_1}(e) \\
& \quad - R_{y_1y_1}(e) - R_{y_2y_2}(e) \\
& = 2 \{ R_{z_1z_1}(e) - R_{z_2z_2}(e) \} \\
& \Rightarrow \text{complementary. Yes!}
\end{align*}
\]

And now \( z_1(n) \) and \( z_2(n) \) are sequences of +1's and -1's.

Whenever:

\[ y_1(n) = 1 \text{ or } -1 \Rightarrow y_2(n) = 0 \]

\[ y_2(n) = 1 \text{ or } -1 \Rightarrow y_1(n) = 0 \]
(T)

$$r_{y_1 y_1 [e]} = r_{x_1 x_1 [e]} + r_{x_2 x_2 [e]}$$

$$+ r_{x_1 x_2 [e+N]} + r_{x_2 x_1 [e-N]}$$

$$r_{y_2 y_2 [e]} = r_{x_1 x_1 [e]} + r_{x_2 x_2 [e]}$$

$$- r_{x_1 x_2 [e+N]} - r_{x_2 x_1 [e-N]}$$

Sum is 2 c s [e] = $r_{y_1 y_1 [e]}$ + $r_{y_2 y_2 [e]$

• The sequences do not overlap and are right up against each other

$y_1 \{n\}$ is unimodular

$y_2 \{n\}$ is unimodular
Prob. 2 (S0114) Solution

(a) \[ Y(z) = \frac{1}{2} + \frac{z^{-1}}{z + \frac{1}{2}} \cdot \frac{z}{z + \frac{1}{2}} = \frac{\frac{1}{2} z + 1}{z + \frac{1}{2}} \]

Zero at \( z = -2 \) \( \{ \text{all-pass} \) \\
pole at \( z = -\frac{1}{2} \) \( \{ \text{filter} \)

(b) \[ \frac{1}{2} \frac{z}{z + \frac{1}{2}} + \frac{z^{-1}}{z + \frac{1}{2}} \]

\( \Longleftrightarrow h[n] = \frac{1}{2} (-\frac{1}{2})^n u[n] \]
\[ \text{note:} \]
\[ \left(-\frac{1}{2}\right)^n \]
\[ = -2 \left(-\frac{1}{2}\right)^n \]
\[ + \left(-\frac{1}{2}\right)^{n-1} u[n-1] \]
\[ = \frac{1}{2} \delta[n] + \left(\frac{1}{2} - 2\right) \left(-\frac{1}{2}\right)^n u[n-1] \]
\[ = \frac{1}{2} \delta[n] + \frac{3}{2} \left(-\frac{1}{2}\right)^n u[n-1] \]
\[ = +2 \left\{ \delta[n] + \left(-\frac{3}{4}\right) \left(-\frac{1}{2}\right)^n u[n] \right\} \]
(c) \( |H(\omega)| = 1 + \omega \)

(d) \( H(\omega) \) is nonlinear

but \( |H(\omega)| = 1 + \omega \), thus:

\( A_0 = 2 \quad A_1 = 1 \quad A_2 = \sqrt{2} \quad A_3 = 3 \)

amplitudes of the sinewaves are unchanged as they pass thru all-pass filter

BUT phases change =)

\[ y[n] \neq x[n] \]

(e) \( x[n] = \frac{1}{\rho} \{ \delta[n] + (\rho^2 - 1) \rho^n u[n] \} \)

\( \Rightarrow \) all-pass signal

\( R_{xx}[\ell] = \delta[\ell] \quad R_{yy}[\ell] = h[\ell] \ast R_{xx}[\ell] \)

(f) \( R_{yx}[\ell] = h[\ell] \)

\( = h[\ell] = 2 \{ \delta[\ell] - \frac{3}{4} \left( -\frac{1}{2} \right)^n u[n] \} \)
\[ (x) = \begin{cases} 1 & x = 0 \\ 0 & x = 1 \end{cases} \]

\[ Y = \begin{cases} 1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases} \]

\[ f(x) = \begin{cases} 1 & x \leq 0 \\ 0 & x > 0 \end{cases} \]