NAME:

EE538 Digital Signal Processing I — Fall 2019
Exam 1 — Friday, Sept. 27, 2019

Cover Sheet

Write your name on this and every page
Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Allowed one double-sided 8.5 x 11 handwritten or typed crib sheet
Calculators NOT allowed.
This test contains **TWO** problems.
Show your work in the space provided for each problem.
Clearly label your work and answer for each part.
You must show all work for each problem to receive full credit.
Always simplify your answers as much as possible.

Prob. No.	**Topic(s)**	**Points**
1.	Autocorrelation and Cross-Correlation and their Related Properties	50
2.	Frequency Response of LTI Systems Pole-Zero Diagrams DT Autocorrelation, Cross-Correlation and their Related Properties	50

Problem 1.[50 pts] We say that sequences $x_1[n]$ and $x_2[n]$ are complementary if their respective autocorrelation sequences sum to a scalar multiple of a Delta function:

$$r_{x_1x_1}[\ell] + r_{x_2x_2}[\ell] = c\,\delta[n]$$

where c is a real-valued, positive constant and the autocorrelations may be expressed as

$$r_{x_ix_i}[\ell] = x_i[\ell] * x_i^*[-\ell] \qquad i = 1, 2$$

For all parts of this problem, you are given $x_1[n]$ and $x_2[n]$ are complementary sequences, both of length N. Answer "Yes" or "No" and then justify your answers through analysis and using the properties of autocorrelation, which need to be cited. Most of the credit is weighted on your justification.

(a) Determine if $y_1[n]$ and $y_2[n]$ are complementary.

$$y_1[n] = x_1[n] \qquad y_2[n] = x_2[n - n_o] \tag{1}$$

(b) Determine if $y_1[n]$ and $y_2[n]$ are complementary.

$$y_1[n] = x_1^*[-n] \qquad y_2[n] = x_2[n] \tag{2}$$

(c) Determine if $y_1[n]$ and $y_2[n]$ are complementary.

$$y_1[n] = e^{j\omega_o n} x_1[n] \qquad y_2[n] = e^{j\omega_o n} x_2[n] \tag{3}$$

(d) (i) Determine if $y_1[n]$ and $y_2[n]$ are complementary. (ii) assume $x_1[n]$ and $x_2[n]$ are sequences of +1's and -1's. Are $y_1[n]$ and $y_2[n]$ also sequences of +1's and -1's.?

$$y_1[n] = \frac{1}{2}(x_1[n] + x_2[n]) \qquad y_2[n] = \frac{1}{2}(x_1[n] - x_2[n]) \tag{4}$$

(e) (i) Determine if $z_1[n]$ and $z_2[n]$ are complementary, where $y_1[n]$ and $y_2[n]$ are defined in part (d). (ii) assume $x_1[n]$ and $x_2[n]$ are sequences of +1's and -1's. Are $z_1[n]$ and $z_2[n]$ also sequences of +1's and -1's.?

$$z_1[n] = y_1[n] + y_2[n] \qquad z_2[n] = y_1[n] - y_2[n] \tag{5}$$

(f) (i) Determine if $y_1[n]$ and $y_2[n]$ are complementary. (ii) assume $x_1[n]$ and $x_2[n]$ are sequences of +1's and -1's. Are $y_1[n]$ and $y_2[n]$ also sequences of +1's and -1's.? Recall N is the length of both $x_1[n]$ and $x_2[n]$.

$$y_1[n] = x_1[n] + x_2[n - N] \qquad y_2[n] = x_1[n] - x_2[n - N] \tag{6}$$

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Problem 2.[50 pts] Consider the DT System characterized by the difference equation

$$\text{System 1:} \quad y[n] = -\frac{1}{2}\, y[n-1] + \frac{1}{2} x[n] + x[n-1] \tag{7}$$

(a) Determine the Transfer Function $H(z)$ for this system and plot the pole-zero diagram.

(b) Write a simple expression for the impulse response of the system, $h[n]$.

(c) Plot the magnitude of the frequency response $|H(\omega)|$ over $-\pi < \omega < \pi$.

(d) Determine the overall output $y[n]$ of the System when the input is the sum of infinite-length sinewaves below

$$x[n] = 2 + e^{j\frac{\pi}{2}n} + \sqrt{2}e^{-j\frac{\pi}{2}n} + 3e^{j\pi n} \tag{8}$$

NOTE 1: Just determine the real-valued, positive amplitudes $A_i, i = 0, 1, 2, 3$ in the expression below. You do NOT need to determine the phase values $\phi_i, i = 0, 1, 2, 3$.
QUESTION: Does the output equal to the input, $y[n] = x[n]$? Explain your answer.

$$y[n] = A_0 e^{j\phi_0} + A_1 e^{j\phi_1} e^{j\frac{\pi}{2}n} + A_2 e^{j\phi_2} e^{-j\frac{\pi}{2}n} + A_3 e^{j\phi_3} e^{j\pi n} \tag{9}$$

(e) Determine the autocorrelation of the output, $r_{yy}[\ell] = y[\ell] * y^*[-\ell]$ for the input signal below, with $p = \frac{3}{4}$.

$$x[n] = \frac{1}{p}\left\{\delta[n] \ + (p^2-1)p^n u[n]\right\} \tag{10}$$

(f) Determine the cross-correlation between the output and input, $r_{yx}[\ell] = y[\ell] * x^*[-\ell]$ with the input signal below, with $p = \frac{1}{2}$.

$$x[n] = \frac{1}{p}\left\{\delta[n] \ + (p^2-1)p^n u[n]\right\} \tag{11}$$

(g) Determine the autocorrelation of the output, $r_{yy}[\ell] = y[\ell] * y^*[-\ell]$, with the input signal $x[n]$ below. Write your answer out in sequence form indicating which value corresponds fo $\ell = 0$.

$$x[n] = \{1, 1, 1, -1, 1\} = \delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4]$$

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