Exam 1: Short Version to Keep When You Leave

Problem 1. [40 points]

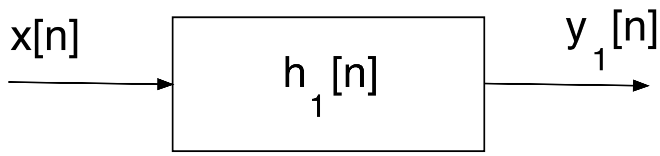
(a) **Note:** System 1 *and* System 2 defined in parts (b) and (c), respectively, are NOT in parallel, but they do have the same input equal to $x[n]$ below with $p = \frac{3}{4}$.

$$x[n] = \frac{1}{p}\left\{\delta[n] \ + (p^2-1)p^n u[n]\right\} \tag{1}$$

Determine & plot the autocorrelation sequence $r_{xx}[\ell]$ for $x[n]$ defined above with $p = \frac{3}{4}$.

(b) Consider System 1 below:

System 1: $\quad y_1[n] = x[n] + x[n-1] - x[n-2] + x[n-3]$



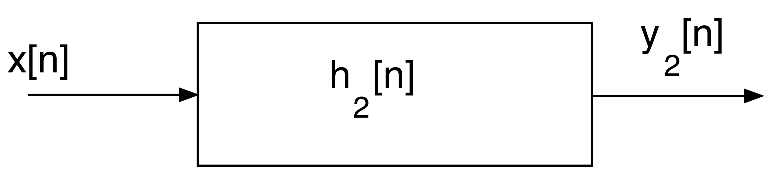
(i) Determine the impulse response $h_1[n]$. You can write it in sequence form.

(ii) Determine the autocorrelation of the impulse response. Do a stem plot of $r_{h_1h_1}[\ell]$.

(iii) For the input signal in Equation (1) above with $p = \frac{3}{4}$, determine the autocorrelation of the output $y_1[n]$. Do a stem plot of $r_{y_1y_1}[\ell]$.

(c) Consider System 2 below:

System 2: $\quad y_2[n] = x[n] + x[n-1] + x[n-2] - x[n-3]$



(i) Determine the impulse response $h_2[n]$. You can write it in sequence form.

(ii) Determine the autocorrelation of the impulse response. Do a stem plot of $r_{h_2h_2}[\ell]$.

(iii) Determine the autocorrelation of the output $y_2[n]$. Do a stem plot of $r_{y_2y_2}[\ell]$.

(d) Using your answers from parts (b) and (c), do a stem plot of $r[\ell] = r_{y_1y_1}[\ell] + r_{y_2y_2}[\ell]$.

Problem 2. [20 points] System 1 and System 2 are as defined in parts (b) and (c) of Problem 1, respectively, but now they are connected in parallel, such that the overall output is

$$y[n] = y_1[n] + y_2[n]$$

(a) Determine the overall response $h[n]$ of the system. Do a stem plot of $h[n]$.

(b) Plot the magnitude of the frequency response of the system $|H(\omega)|$ over $-\pi < \omega < \pi$.

(c) Determine a closed-form expression for the output $y[n]$ when the input is $x[n]$ below.

$$x[n] = 1 + (j)^n + (-j)^n + (-1)^n$$

Problem 3. [40 points]

(a) The autocorrelation of $x[n]$ may be computed in terms of convolution as

$$r_{xx}[\ell] = x[\ell] * x^*[-\ell]$$

Which of the three main properties of convolution discussed in class is key to proving that the autocorrelation of $y[n] = x^*[-n]$ is the same as the autocorrelation of $x[n]$. Write your one-word answer directly below.

(b) Is the convolution of two all-pass filters/sequences also an all-pass filter/sequence? Prove your answer with a short mathematical analysis in the space provided on the next few blank pages.

(c) Let $x[n]$ be the input signal described below with $p = \frac{1}{2}$.

$$x[n] = \frac{1}{p}\left\{\delta[n] + (p^2 - 1)p^n u[n]\right\} \tag{2}$$

This signal is input to the system below described the difference equation below

$$y[n] = \frac{2}{3}y[n-1] - \frac{2}{3}x[n] + x[n-1]$$

(i) Draw the pole-zero diagram for this system. Is the system stable?

(ii) Determine the autocorrelation of the output $y[n]$. Do a stem plot of $r_{yy}[\ell]$.

(d) For the same input signal and same system described in part (c), determine the cross-correlation $r_{yx}[\ell]$ between the input and output. Just provide a closed-form expression for $r_{yx}[\ell]$; you don't need to plot it.