

Cover Sheet

Test Duration: 60 minutes.

Coverage: Chapters 1-5.

Open Book but Closed Notes.

Calculators NOT allowed.

This test contains **two** problems.

Show your work in the space provided for each problem.

You must show all work for each problem to receive full credit.

Always simplify your answers as much as possible.

Prob. No.	Topic(s)	Points
1.	DT Autocorrelation, Cross-Correlation Correlation in terms of Convolution	60
2.	LTI Systems: Properties, Transfer Functions, Frequency Response	40

Problem 1. [60 points]

- (a) Consider $x[n]$ to be a real-valued sequence with autocorrelation $r_{xx}[\ell]$. Express the autocorrelation sequence $r_{yy}[\ell]$ for $y[n] = x[-n]$ in terms of $r_{xx}[\ell]$. Be sure to show clearly how you arrived at your answer.

- (b) Determine and plot the autocorrelation sequence $r_{xx}[\ell]$ for $x[n]$ defined below with $p = \frac{1}{2}$. *Note:* This sequence is used in parts (c) thru (f).

$$x[n] = \frac{1}{p} \left\{ \delta[n] + (p^2 - 1)p^n u[n] \right\} \quad (1)$$

- (c) Determine and plot the autocorrelation sequence $r_{zz}[\ell]$ for $z[n]$ defined in terms of $x[n]$ in Eqn. (1) as below.

$$z[n] = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{\sqrt{2}}\right)} x[n]$$

- (d) Determine and plot the autocorrelation sequence $r_{zz}[\ell]$ for $z[n]$ defined in terms of $x[n]$ in Eqn. (1) below.

$$z[n] = x[2 - n]$$

- (e) Consider a simple radar example where there are two echoes such that the received signal may be expressed in terms of $x[n]$ in Eqn. (1) as

$$y[n] = x[n - 3] + \frac{1}{2}x[n - 5] \quad (2)$$

Compute and plot the cross-correlation sequence $r_{yx}[\ell]$ given the input sequence $x[n]$ defined in Eqn. (1) above.

- (f) Recall the simple radar example described by Eqn (2). Compute and plot the **auto-correlation** sequence for the **output** $r_{yy}[\ell]$ given the input sequence $x[n]$ defined in Eqn. (1). Can echo delays be determined from the autocorrelation of the output $r_{yy}[\ell]$?

Problem 2. [40 points] Consider the causal DT LTI system described the difference equation below which is used for all four parts of this problem.

$$y[n] = j y[n - 1] + x[n] - x[n - 4]$$

- (a) Determine the transfer function $H(z)$ for this system and plot the pole-zero diagram. Show the region of convergence.

- (b) Determine the impulse response of the system $h[n]$. You can either list the value of $h[n]$ for each n or write a closed-form expression for $h[n]$ that works for all n .

- (c) Determine a closed-form expression for the frequency response of the system $H(\omega)$ equal to the DTFT of $h[n]$. Plot the magnitude $|H(\omega)|$ over $-\pi < \omega < \pi$. You can provide a rough sketch, but you need to clearly indicate the frequencies where $|H(\omega)| = 0$ and the frequency where $|H(\omega)|$ reaches its peak value.

- (d) Determine a closed-form expression for the output $y[n]$ when the input is the following sum of sinewaves turned-on forever:

$$x[n] = 1 + 2(-j)^n + 3(j)^n + 4(-1)^n$$