Exam 1 Solution — Fall 2010

Prob. 1 (a) $x[n] = \{1, 1, -1, 1\}$

$\ell = 0$: $r_{xx}[0] = 3(1)^2 + (-1)^2 = 4$

$\ell = 1$:
$$\begin{array}{l} \{1, 1, -1, 1\} \\ \times \quad \{1, 1, -1, 1\} \\ = \quad 1 - 1 - 1 = -1 \end{array}$$
$r_{xx}[1] = -1 = r_{xx}[-1]$

$\ell = 2$:
$$\begin{array}{l} \{1, 1, -1, 1\} \\ \times \quad \underline{1, 1} \\ -1 + 1 = 0 \end{array}$$
$r_{xx}[2] = r_{xx}[-2] = 0$

$\ell = 3$: $\{1, 1, -1, 1\}$

$1 = 1$ $\quad r_{xx}[3] = r_{xx}[-3] = 1$

$$r_{xx}[\ell] = \{1, 0, -1, 4, -1, 0, 1\}$$

See plots later.

1 (b) $z[n] = x[n-4]$

From "Key Problem" on Exam 1 F'06,

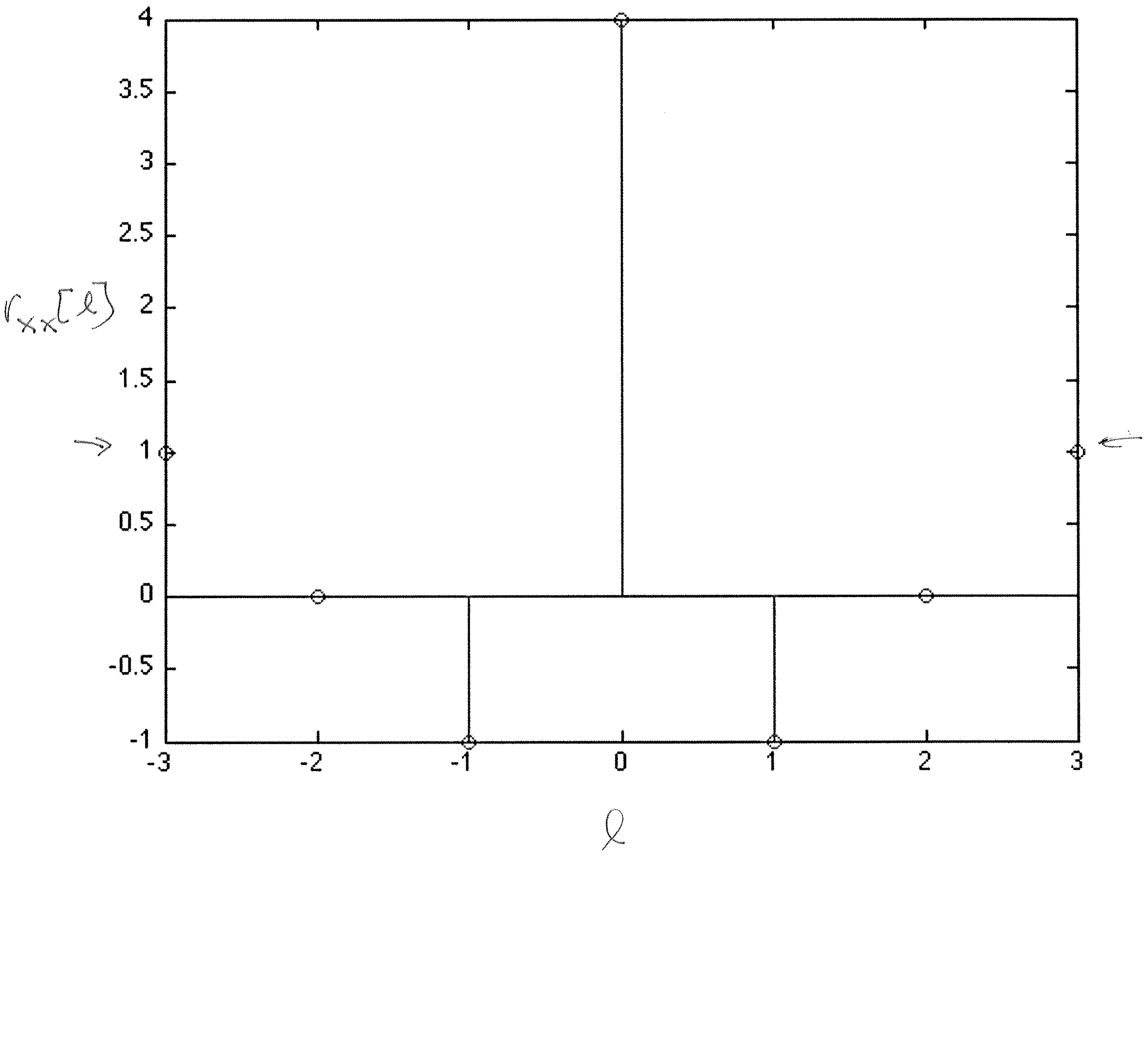
Problem 1 (c): if: $z[n] = x[n - n_0]$

then: $r_{zz}[\ell] = r_{xx}[\ell]$

Stated at least twice in class that time-shift does not change autocorrelation

$$r_{zz}[\ell] = \{1, 0, -1, 4, -1, 0, 1\}$$

Plot for Prob. 1 (a)
same plot for Prob. 1 (b)



Exam 1 Solution Fall 2010

Prob. 1 (c) ②

$$z[n] = (-1)^n x[n]$$
$$= e^{j\pi n} x[n]$$

From "Key Problem" on Exam 1, F'06 ;
Prob. 1, part (d):

$$\text{If: } z[n] = e^{j(\omega_0 n + \theta)} x[n]$$

$$\text{then: } r_{zz}[\ell] = e^{j\omega_0 \ell} r_{xx}[\ell]$$

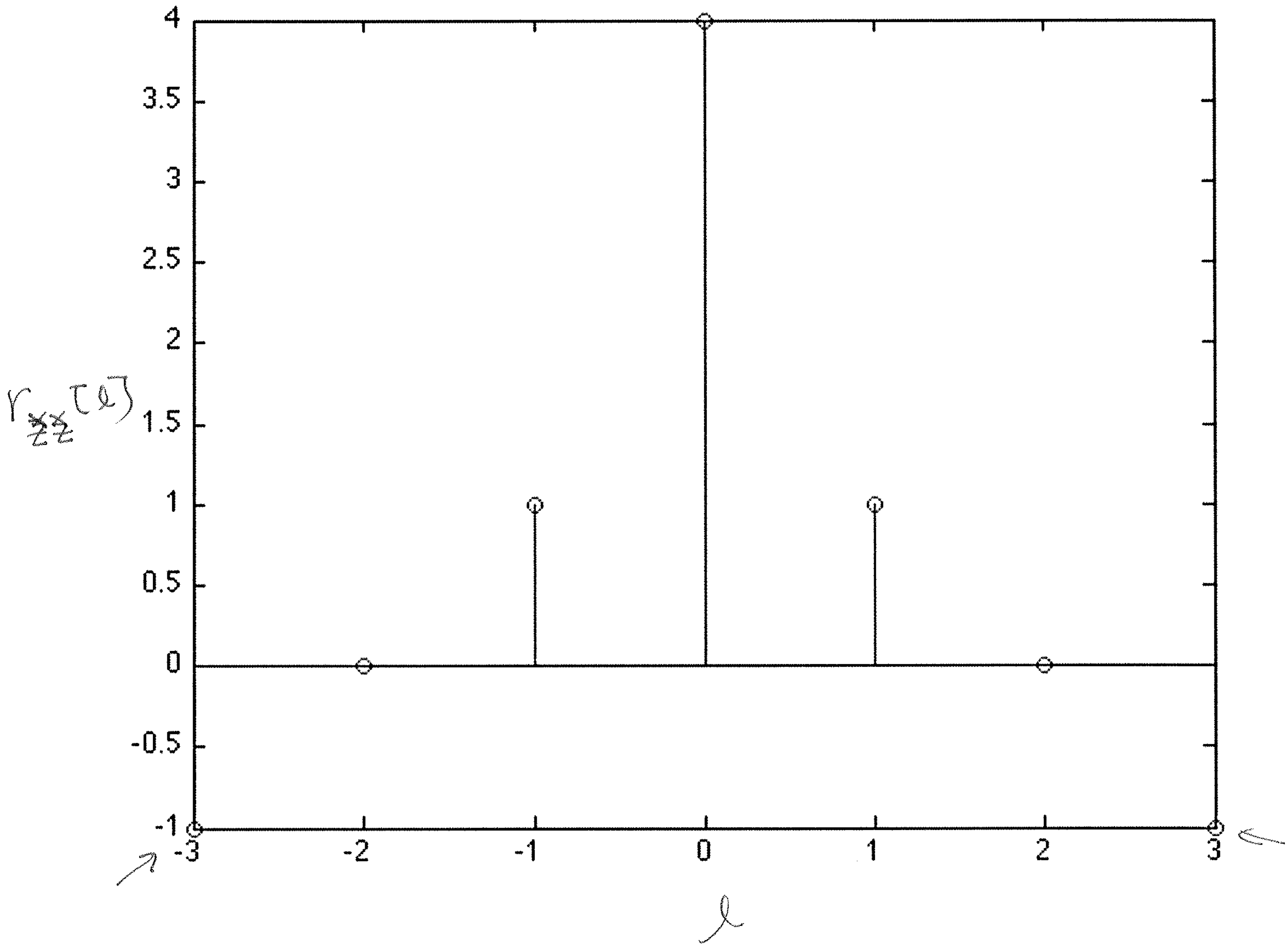
Stated at least twice in-class, if you multiply a signal by a sinewave, the new autocorrelation is the old one multiplied by a sinewave at the same frequency but the phase is gone:

$$\text{Thus: } r_{zz}[\ell] = e^{j\pi\ell} r_{xx}[\ell]$$
$$= (-1)^{\ell} r_{xx}[\ell]$$

$$r_{zz}[\ell] = \{-1, 0, 1, \underset{\uparrow}{4}, 1, 0, -1\}$$

This theory applies to the next part as well.

Plot for Prob. 1 (c)



Prob. 1 (d) $z[n] = e^{j(\frac{\pi}{2}n + \frac{\pi}{3})n} x[n]$

3

$r_{zz}[\ell] = e^{j\frac{\pi}{2}\ell} r_{xx}[\ell]$

$= (j)^{\ell} r_{xx}[\ell]$

$$
\begin{array}{ll}
r_{xx}[\ell]: & \{1, 0, -1, 4, -1, 0, 1\} \times \\
(j)^{\ell}: & \underline{\{j, -1, -j, 1, j, -1, -j\}} \\
r_{zz}[\ell]: & \{j, 0, j, \underset{\uparrow}{4}, -j, 0, -j\}
\end{array}
$$

could plot real & imaginary parts separately — not done here

1)-(e): Parts (e) and (f) of this problem make use of the updated handout posted at the course website (and emailed as well) entitled "Autocorrelation Properties: Proofs"

See page 8, where we showed that for the two-target case:

If: $y[n] = T_1 x[n-D_1] + T_2 x[n-D_2]$

then: $r_{yx}[\ell] = T_1 r_{xx}[\ell - D_1] + T_2 r_{xx}[\ell - D_2]$

IN ADDITION, a study problem was from Exam 1, Fall 2008, Problem 3:

$x[n] = \{\underset{\uparrow}{4}, 2, 1\}$

$y[n] = 2x[n-2] + x[n-6]$

Prob. 1 (e): (4)

$$y[n] = x[n-4] - x[n-8]$$

$$r_{yx}[\ell] = r_{xx}[\ell-4] - r_{xx}[\ell-8]$$

$$r_{yx}[\ell] = \{0, 1, 0, -1, 4, -1, 0, 1\}$$

$\underset{\ell=0}{\uparrow}$ minus $- \{0, 1, 0, -1, 4, -1, 0, 1\}$

$$r_{yx}[\ell] = \{\underset{\uparrow}{0}, 1, 0, -1, 4, -2, 0, 2, -4, 1, 0, -1\}$$

Prob. 1 (f): $y[n] = x[n-4] + x[n-6]$

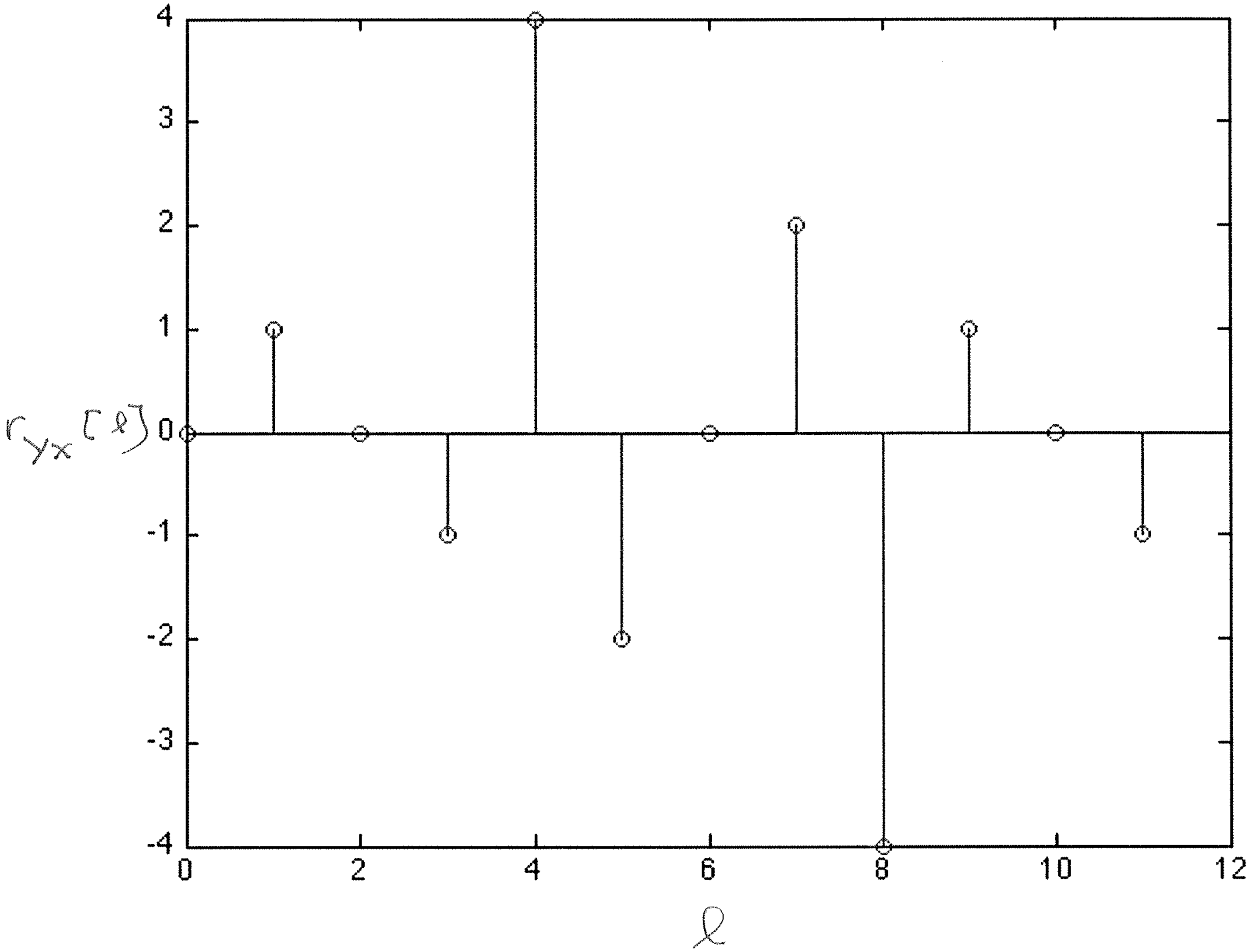
$$r_{yx}[\ell] = r_{xx}[\ell-4] + r_{xx}[\ell-6]$$

$$r_{yx}[\ell] = \{\underset{\uparrow}{0}, 1, 0, -1, 4, -1, 0, 1\}$$

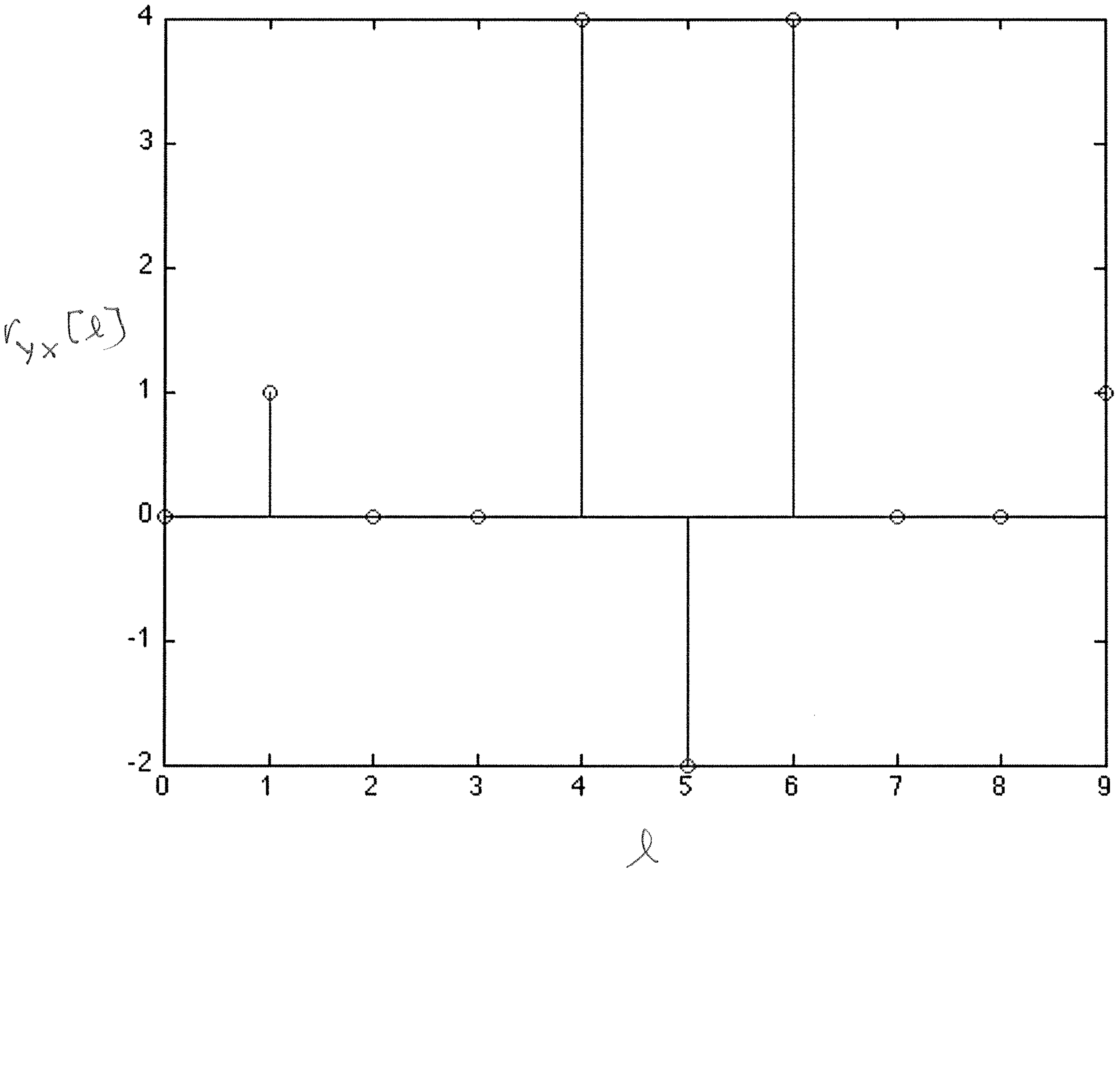
$\ell=0$ $+ \{0, 1, 0, -1, 4, -1, 0, 1\}$

$$r_{yx}[\ell] = \{\underset{\substack{\uparrow \\ \ell=0}}{0}, 1, 0, 0, 4, -2, 4, 0, 0, 1\}$$

Plot for Prob. 1 (e)



Plot for Prob. 1 (f)



Problem 2 See OFDM Example at course web site for details.

We throw away the $L=3$ values at the beginning and the $L-1=2$ values at the end, and retain only the $N=4$ full overlap values:

$$\tilde{y}[n] = \{\underset{\uparrow}{2-2j}, 2+2j, -2+2j, 2+2j\}$$

Need to evaluate $H(\omega) = 1 + j e^{-j2\omega}$ at $N=4$ frequencies. Using Z-Transform:

$$\omega = 0 \Rightarrow z = 1 \Rightarrow H(z)\Big|_{z=1} = 1 + j(1)^{-2} = 1+j$$

$$\omega = \frac{\pi}{2} \Rightarrow z = j \Rightarrow H(z)\Big|_{z=j} = 1 + j(j)^{-2} = 1-j$$

$$\omega = \pi \Rightarrow z = -1 \Rightarrow H(z)\Big|_{z=-1} = 1 + j(-1)^{-2} = 1+j$$

$$\omega = \frac{3\pi}{2} \Rightarrow z = -j \Rightarrow H(z)\Big|_{z=-j} = 1 + j(-j)^{-2} = 1-j$$

Need to multiply $\tilde{y}(n)$ by the complex conjugate of each of the $N=4$ sinewaves and sum, then divide by $H\left(k\frac{2\pi}{4}\right)$ and 4

$$b_0 = \frac{(1)(2-2j)+(1)(2+2j)+(1)(-2+2j)+(1)(2+2j)}{4\ (1+j)}$$

$$= 1$$

$$b_1 = \frac{(1)\ (2-2j)+(j)^*(2+2j)+(-1)(-2+2j)+(-j)^*(2+2j)}{4\ (1-j)}$$

$$= 1$$

$$b_2 = \frac{(1)(2-2j)+(-1)(2+2j)+(1)(-2+2j)+(-1)(2+2j)}{4\,(1+j)}$$

$$= -1$$

$$b_3 = \frac{(1)(2-2j)+(-j)^*(2+2j)+(-1)(-2+2j)+(j)^*(2+2j)}{4\,(1-j)}$$

$$= 1$$

$$\{b_0, b_1, b_2, b_3\} = \{1, 1, -1, 1\}$$