EE538 Digital Signal Processing I **Fall 2010**
Exam 1 **Friday, Oct. 1, 2010**

Cover Sheet

Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems.
All work should be done in blue books.
You must show all work for each problem to receive full credit.
Do **not** return the exam itself; just your blue book.

Prob. No.	**Topic(s)**	**Points**
1.	LTI Systems: Properties, Transfer Functions, Frequency Response	55
2.	DT Autocorrelation, Cross-Correlation Correlation in terms of Convolution	45

Problem 1. [55 points] Be sure to label each part carefully in your blue book.

(a) Determine and plot the autocorrelation sequence $r_{xx}[\ell]$ for $x[n]$ below.

$$x[n] = \{1, 1, -1, 1\} \tag{1}$$

(b) Determine and plot the autocorrelation sequence $r_{zz}[\ell]$ for $z[n]$ defined in terms of $x[n]$ in Eqn. (1) as

$$z[n] = x[n-4]$$

(c) Determine and plot the autocorrelation sequence $r_{zz}[\ell]$ for $z[n]$ defined in terms of $x[n]$ in Eqn. (1) as

$$z[n] = (-1)^n x[n]$$

(d) Determine the autocorrelation sequence $r_{zz}[\ell]$ for $z[n]$ defined in terms of $x[n]$ in Eqn. (1) below. For this part, you need only write out the autocorrelation sequence; you don't need to plot it.

$$z[n] = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{3}\right)} x[n]$$

(e) Consider a simple radar example where there are two echoes such that the received signal may be expressed in terms of $x[n]$ in Eqn. (1) as

$$y[n] = x[n-4] - x[n-8]$$

Compute and plot the cross-correlation sequence $r_{yx}[\ell]$ given the input sequence $x[n]$ defined in Eqn. (1) above.

(f) Repeat for the case where the received signal may be expressed in terms of $x[n]$ as

$$y[n] = x[n-4] + x[n-6]$$

Compute and plot the cross-correlation sequence $r_{yx}[\ell]$ given the input sequence $x[n]$ defined in Eqn. (1) above.

Problem 2. [45 points]

Consider $N = 4$ information symbols, denoted $\{b_0, b_1, b_2, b_3\}$. A sequence of length $N = 4$ is created as a sum of sinewaves as indicated below.

$$x[n] = \sum_{k=0}^{3} b_k s_k[n] \quad \text{where:} \quad \begin{array}{l} s_0[n] = e^{j0\frac{2\pi}{4}n}\{u[n] - u[n-4]\} = \{1, 1, 1, 1\} \\ s_1[n] = e^{j1\frac{2\pi}{4}n}\{u[n] - u[n-4]\} = \{1, j, -1, -j\} \\ s_2[n] = e^{j2\frac{2\pi}{4}n}\{u[n] - u[n-4]\} = \{1, -1, 1, -1\} \\ s_3[n] = e^{j3\frac{2\pi}{4}n}\{u[n] - u[n-4]\} = \{1, -j, -1, j\} \end{array}$$

The last $L = 3$ values of $x[n]$ are appended to the beginning to form a sequence of length 7 as

$$x[n] = \{x[-3], x[-2], x[-1], x[0], x[1], x[2], x[3]\} \tag{2}$$
$$= \{x[+1], x[+2], x[+3], x[0], x[1], x[2], x[3]\} \tag{3}$$

Next, $x[n]$ is input to a Linear Time-Invariant (LTI) system with an impulse response of length $L = 3$ equal to

$$h[n] = \{h[0], h[1], h[2]\}$$
$$= \{1, 0, j\}$$

to produce an output sequence

$$y[n] = x[n] * h[n]$$

of length equal to $7 + 3 - 1 = 9$.

$$y[n] = \{y[-3], y[-2], y[-1], y[0], y[1], y[2], y[3], y[4], y[5]\} \tag{4}$$
$$= \{2,\ -2,\ 2+2j,\ 2-2j,\ 2+2j,\ -2+2j,\ 2+2j,\ -2j,\ 2j\} \tag{5}$$
$$\tag{6}$$

Here's your task: given $h[n] = \{h[0], h[1], h[2]\} = \{1, 0, j\}$ and

y[n]	y[-3]	y[-2]	y[-1]	y[0]	y[1]	y[2]	y[3]	y[4]	y[5]
=	2	-2	2+2j	2-2j	2+2j	-2+2j	2+2j	-2j	2j

determine the 4 values of the information symbols $\{b_0, b_1, b_2, b_3\}$. You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.